

Math 9

Module 7

PR 1: Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.

PR2: Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.

Focus 1: Use equations to describe and solve problems involving patterns,

Skills Review:

(1) Substituting values into equations:

Ex 1: For the equation $P=4x$, find the value of P when $x = 6$.

Substitute the x with the number 6

$$\begin{aligned} \text{So } P &= 4(6) \\ &= 24 \end{aligned}$$

Ex 2: For the equation $d = 15t + 7$, find the value of d when $t = 8$

Substitute the t with the number 8

$$\begin{aligned} \text{So } d &= 15(8) + 7 && \text{**** Follow order of operations ***} \\ d &= 120 + 7 \\ d &= 127 \end{aligned}$$

Ex 3: For $y = 4x + 6$, find the value of x when $y = 34$

Substitute $y = 34$ then solve the equation $34 = 4x + 6$

$$\begin{aligned} 34 - 6 &= 4x + 6 - 6 \\ 28 &= 4x \\ \underline{28} &= \underline{4x} \\ 4 & \quad 4 \\ 7 &= x \end{aligned}$$

TRY THESE:

- (1) Find c for the equation $c= 6a -2$, when $a = 12$
- (2) Find d for the equation $d = 5-7a$, when $a = -2$
- (3) Find y for the equation $y = \frac{1}{2} x + 8$ when $x = 14$
- (4) Find t for the equations $d = 7t -1$ when $d = 27$

Answers:

(1) $c = 70$ (2) $d = 19$ (3) $y = 15$ (4) $t = 4$

(2) Completing a table of values

Complete the table of values using the equation $y = 2x + 5$

x	y
-2	
-1	
0	
1	
2	

Substitute the values of x into the equation 1 at a time

$$y = 2(-2) + 5 = -4 + 5 = 1$$

$$y = 2(-1) + 5 = -2 + 5 = 3$$

$$y = 2(0) + 5 = 0 + 5 = 5$$

$$y = 2(1) + 5 = 2 + 5 = 7$$

$$y = 2(2) + 5 = 4 + 5 = 9$$

The completed table looks like

x	y
-2	1
-1	3
0	5
1	7
2	9

Do you see a pattern in the table?

How is this related to the equation $y = 2x + 5$?

TRY THIS ONE:

Complete a the table of values for $y = -3x + 10$

x	y
-2	
-1	
0	
1	
2	

Do you see a similar pattern in the table values for $y = -3x + 10$?

Where does the 10 in the equation come from?

Working backwards

Given the table of values we can find the equation if there is a pattern in the table.

x	y
1	6
2	8
3	10
4	12

Look for the pattern

x values go up by 1

y values go up by 2

From the previous examples, we know to write the equation it will look like

$y = \text{---} x + \text{---}$

dependent variable (second column in the table) independent variable (first column in the table)

The number that goes in front of the x in the equation is the amount that the values of y go up or down by in the table (when x values go up by 1). This has to be the same for all y values.

In this case the y goes up by 2 so the equation would be

$$y = 2x + \underline{\quad}$$

To figure out what the number where the $\underline{\quad}$ is would be we substitute values from the table in.

The first pair in the table is $x = 1$ and $y = 6$. Replacing these in the equation would give us

$$y = 2x + \underline{\quad}$$

$$6 = 2(1) + \underline{\quad}$$

$$6 = 2 + \underline{\quad} \quad \text{To make this statement true the value in } \underline{\quad} \text{ is } 4$$

So the equation we were looking for is

$$y = 2x + 4$$

To be sure we can verify using another ordered pair from the table.

Using $x = 3$ and $y = 10$

$$Y = 2x + 4$$

$$10 = 2(3) + 4$$

$$10 = 6 + 4$$

$$10 = 10 \quad \checkmark \quad \text{This works so the equation is correct.}$$

TRY THESE:

(1) Find the equation

m	h
1	21
2	32
3	43
4	54

a) Determine the pattern

b) Make the adjustment

c) State the equation

(2) Find the equation

r	s
5	56
6	70
7	84
8	98

a) Determine the pattern

b) Make the adjustment

c) State the equation

d) Use your equation to determine
the value of "s" when $r = 200$

(3) Find the equation

x	y
1	12
2	9
3	6
4	3

a) Determine the pattern

b) Make the adjustment

c) State the equation

Answers:

1 (c) $h = 11m + 10$

2 (c) $s = 14r - 14$ (d) 2786

3 (c) $y = -3x + 15$

Writing Equations Using Models

EX 1:

Look at each figure is there a pattern?

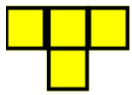


Figure 1

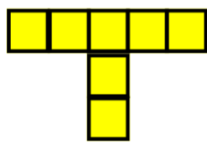


Figure 2

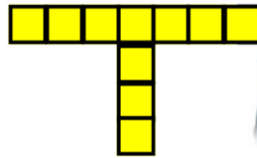


Figure 3

Make a table of values

Figure Number (f)	Number of Squares (n)
1	4
2	7
3	10

Next look for a pattern..

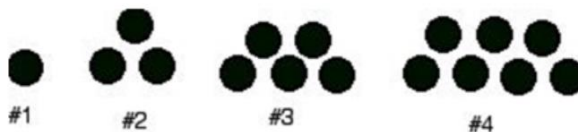
F increases by 1 in the table and n increases by 3

Write the equation

$$n = 3f + 1$$

EX 2:

Write an equation that relates the number of circles, c, to the figure number, f.



TRY IT!!!

*****MAKE A TABLE OF VALUES AND LOOK FOR PATTERNS*****

EX 3:

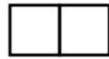


How many people can sit at the tables?
(only one person per edge)

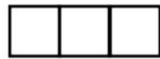
Table Seating



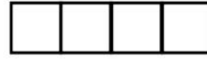
1 Table



2 Tables



3 Tables



4 Tables

How many people can sit
with 100 tables?

HINT: Make a table of values, look for patterns in the table & write the equation.

Then use the table to predict how many people 100 tables can hold.

CHALLENGE: How many tables would be required to seat 100 people?

GET YOUR ANSWERS TO EX 2 & 3 CHECKED NOW!!

Writing an Equation to Represent a Written Pattern

Ex1:



A large water tower holds 15000 liters of water, however during the winter the water tower was damaged and started to leak. This table shows the amount of water every hour after it sprung the leak. The level of water changes at a constant rate.



Time (t) hours	Amount (v) Litres
0	15 000
1	14 800
2	14 600
3	14 400
4	14 200

i) Write an expression for the amount in terms of the time since the water tower began to leak.

Look for a pattern in the table..

t goes up by 1 and v goes down by 200

so the expression would be $15000 - 200t$

ii) Write an equation that relates the amount of water to the time since it started leaking.

The equation would be : $v = 15000 - 200t$

***** Note the difference between writing an expression and an equation*****

iii) How much water in in the water tower after 10 hours?

Use the equation and then substitute 10 in for t

$$v = 15000 - 200t$$

$$v = 15000 - 200(10)$$

$$v = 15000 - 2000$$

$$v = 13000$$

There would be 13000 L of water left in the tank after 10 h.

iv) When will the water tower be empty?

Use the equations and substitute 0 in for v. Then solve the equation.

$$v = 15000 - 200t$$

$$0 = 15000 - 200t$$

$$0 - 15000 = 15000 - 200t - 15000$$

$$-15000 = -200t$$

$$\frac{-15000}{-200} = \frac{-200t}{-200}$$

$$75 = t$$

The water tower will be empty after 75 hours.

Writing an Equation to Represent an Oral Pattern

EX1:


Crazy Cab charges a fixed cost of \$4.50 for each fare plus a rate of \$1.75 for each kilometer travelled.

(a) Write an equation that relates the cost of the fare to the distance driven.

To write an equation variables are required. When we use variables we must always state what the variables we are using represent

Let c represent the cost of the fare and d represent the distance driven.

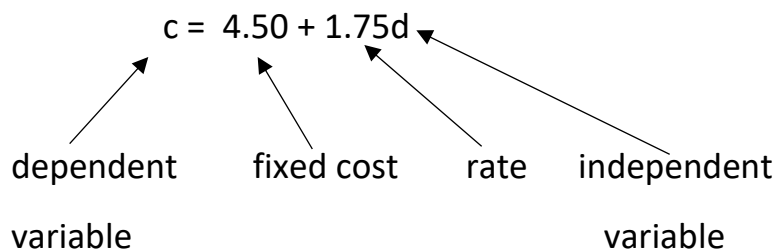
The cost depends on the distance driven so cost is the dependent variable and distance driven is the independent variable.

 Look up the definitions for dependent and independent variable in your textbook and copy these into your notes.

\$4.50 is a fixed amount- The cab charges this amount to everyone no matter how far they travel

The \$ 1.75 is the rate. The cab will charge this amount for each km driven.

So the equation will be



(b) What would the cost of a fare of 12 km be?

Use the equation and substitute 12 in for d.

$$c = 4.50 + 1.75d$$

$$c = 4.50 + 1.75(12)$$

$$c = 4.50 + 21$$

$$c = 25.50$$

It would cost \$25.50 to travel 12 km.

TRY THIS:

Sarina babysits on the weekend to make some spending money. She charges a fixed rate of \$12 and \$8/hr.

(A) Write an equation to relate the amount of money Sarina earns to the number of hours she babysits.

(B) If Sarina babysits for 4 hours, how much money will she make?

(C) On one Saturday Sarina earned \$72. How long did she babysit that day?



Do Practice Questions p.159-161 #4 to 21

Do Section 4.1 Quiz

Focus 2: Analyze the graph of a linear relation.

A Blast From the Past



Complete Plotting Points on Co-ordinate Grid Worksheet and have it corrected.

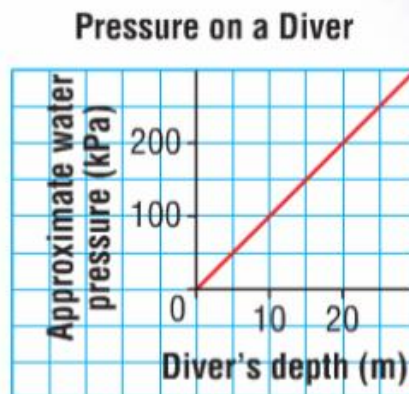
If you need a refresher on plotting points, watch the U Tube video:

<https://www.youtube.com/watch?v=s7NKLWXkEEE>

Linear Relations

When a scuba diver goes under water, the weight of the water exerts pressure on the diver.

Diver's Depth (m)	Approximate Water Pressure (kiloPascals)
0	0
5	50
10	100
15	150
20	200



What pattern do you see in the table?

What pattern do you see in the graph?

What do these patterns tell us about the relationship between depth and water pressure?

Investigate:

A local phone company offers a cell phone plan that has a fixed cost of \$30 per month plus an additional cost of \$0.20 for each message sent.

(a) Represent the relationship between cost (c) and number of messages sent (n) as an equation.

(b) Represent the relationship between cost(c) and the number of messages sent(n) in a table of values.

(c) Graph the relationship.

(d) What patterns do you see in the table and the graph ?

ANSWER:

(a) The cost of sending the messages depends on the number of messages sent so C is the dependent variable & n is the independent variable.

30 is a fixed cost. This amount must be paid even if no texts are sent.

0.20 is the rate or the cost for each message.

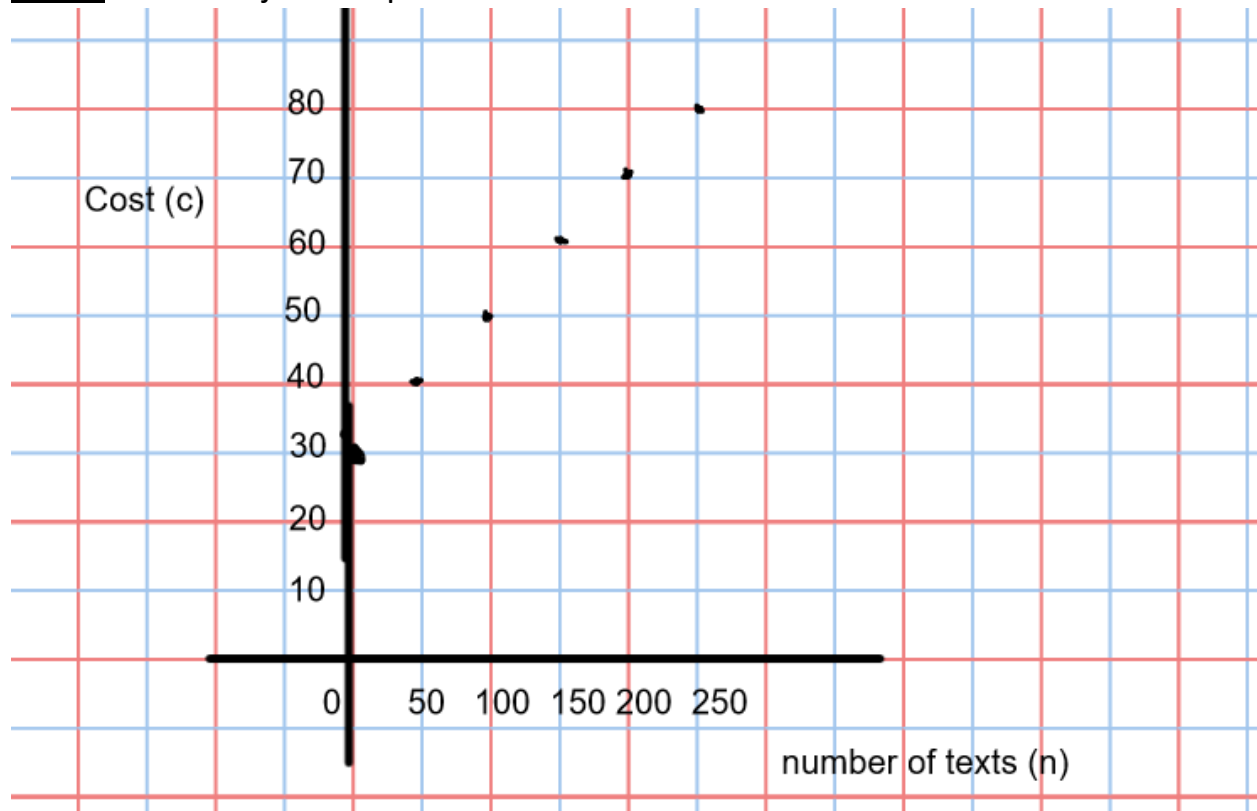
So our equation is : $c = 0.20n + 30$ (Refer back to focus 1 if necessary)

(b)

Number of texts (n)	Cost (c)
0	30
50	40
100	50
150	60
200	70
250	80

(c)

Note: We don't join the points because we can't send a fraction of a text



message.

(d) In the table, the values of n increase by 50 and the values of c increase by 10. These are constant amounts.

On the graph the points form a straight line. The relationship between the number of texts sent and the cost is linear.

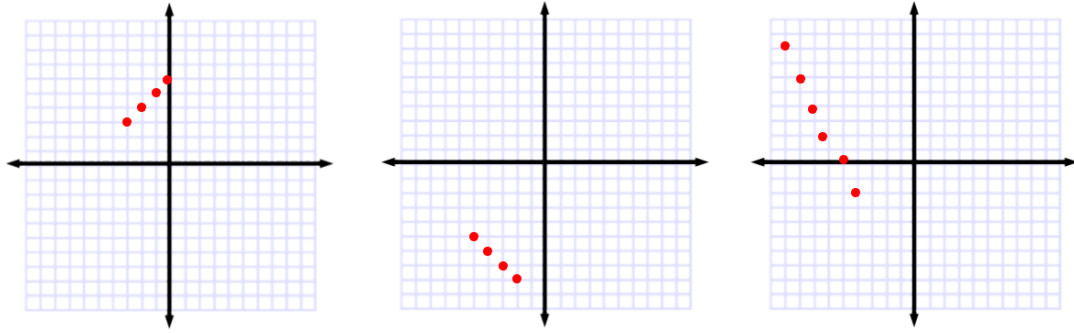
Identifying a Linear Relation

➡ When the graph of a linear relation is a straight line, we have a linear relation

➡ In a table of values, a relation is linear if a constant change in the independent variable produces a constant change in the dependent variable.

Example 1:

Can you identify the linear relations?



Example 2:

(a)

Can you identify the linear relation?

x	y
6	35
7	30
8	25
9	20

x	y
6	35
7	37
8	39
9	42

(b)

Can you identify the linear relation?

x	y
0	18
1	21
2	24
3	27

x	y
5	2
6	4
7	8
8	16



Do questions p. 170-171 # 4 to 9

Graphing a Linear Relation from an Equation:

To graph a linear relation from an equation

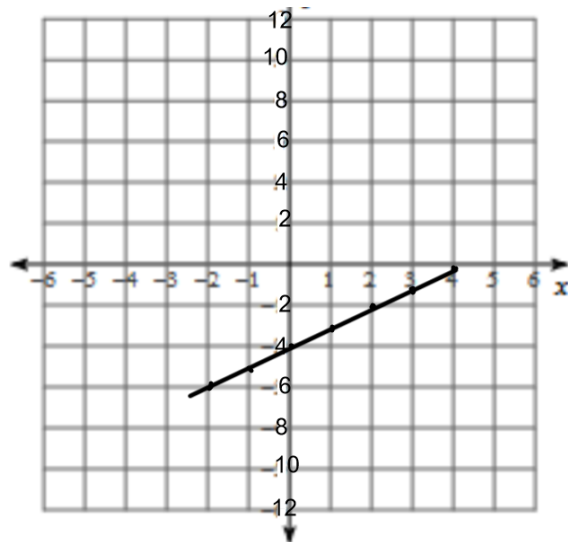
- Make a table of values
- Graph the relation. Join the points on the graph only if all the values may be Real Numbers (no fractions or decimals)
- Check to see if the pattern in the table matches the pattern on your graph. (Is the relation linear?)

Examples:

Graph:

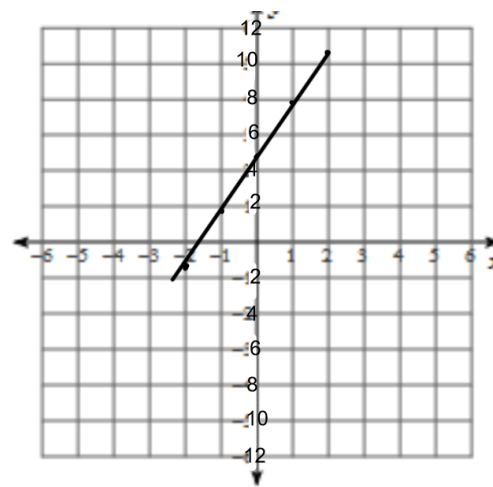
(1) $y = x - 4$

x	y
-2	-6
-1	-5
0	-4
1	-3
2	-2
3	-1



(2) $y = 3x + 5$

x	y
-2	-1
-1	2
0	5
1	8
2	11



TRY THESE:

Make a table of values and graph:

(1) $y = -2x + 3$

(2) $y = 4 - 3x$

Get your answers checked before proceeding.

Solving Problems Using a Linear Relation

Example:

The student council is planning to hold a dance. The profit in dollars is 4 times the number of students who attend, minus \$200 for the cost of the music.

- a) Write an equation that relates the profit to the number of students who attend.
- b) Create a table of values for this relation.
- c) Graph the data in the table. Does it make sense to join the points? Explain.
- d) How many students have to attend to make a profit?

Solution:

a) Define the variables to be used

Let p represent how much profit is made.

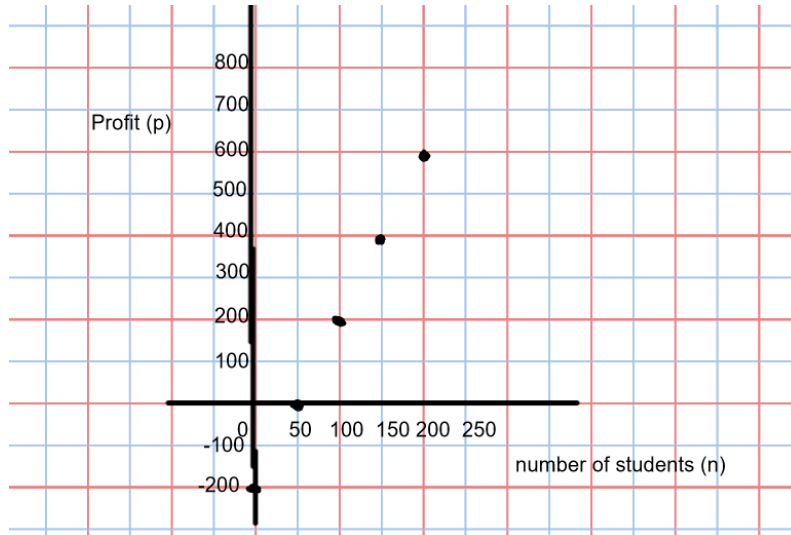
Let n represent the number of students that attend the dance.

So $p = 4n - 200$

b)

n	p
0	-200
50	0
100	200
150	400
200	600

c)



Note: The points should not be joined. Even though points do exist between those plotted on the graph, not all points are permitted. For example it would be possible to have 75 students attend the dance but not 75.5 students.

d) There must be more than 50 students attend to make a profit. The least amount of students that must attend to make a profit is 51.



Do practice questions p. 171-173 #10 to 16

Do Assignment : BLM 4.2 – Pass In

Complete Quiz #2

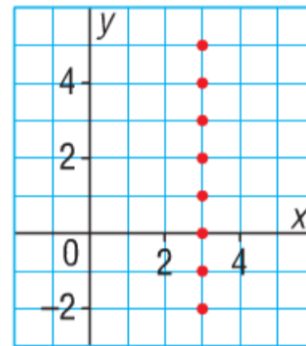
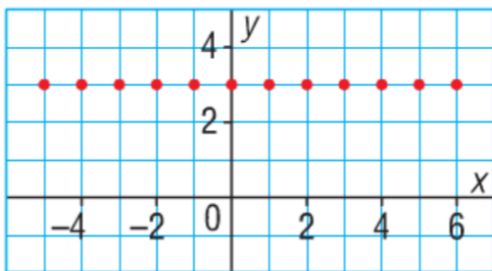
Focus 3: Recognize the equations of horizontal, vertical and oblique lines, and graph them.

Horizontal and Vertical Lines

HORIZONTAL

VS

VERTICAL



$$y = 3$$

no x

$$x = 3$$

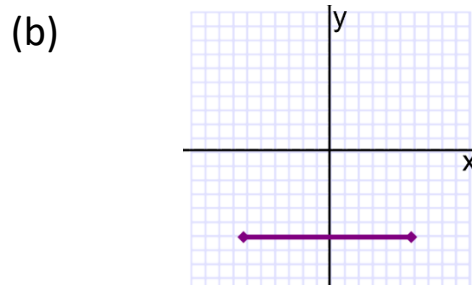
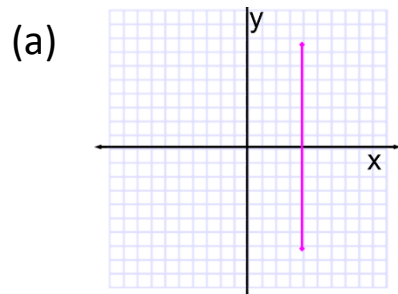
no y

For every 'x' value,
y will always equal 3.

For every 'y' value,
x will always equal 3.

TRY THESE:

Give the equation for each of the following lines.



Hint: Think about which axis it crosses.

- Crosses x axis, equation is $x =$
- Crosses y axis, equation is $y =$

(c) Draw a graph and describe each of the following equations

(i) $y = -2$ (ii) $x = 4$ (iii) $x = \frac{1}{2}$ (iv) $2y - 6 = 0$

HINT: For equations like (iv) the equation must be rearranged so that the variable is by itself on one side of the equation. The steps for rearranging equations are the same as those used for solving equations in Module 6. Review these if necessary.

Examples:

(1) $x + 7 = 0$

$$x + 7 - 7 = 0 - 7$$

$$x = -7$$

(2) $3y - 9 = 0$

$$3y - 9 + 9 = 0 + 9$$

$$3y = 9$$

$$\underline{3y} = \underline{9}$$

$$\frac{3}{3} \quad \frac{9}{3}$$

$$y = 3$$

OBLIQUE LINES

Oblique lines are lines that are **slanted**. All lines can be classified as vertical, horizontal or oblique.

We can identify the type of line on the graph by determining what the line looks like.

To identify the type of line from the equation we take note of the variables in the equation. If the linear equation has only an x variable it is vertical, only a y variable it is horizontal and if it has both an x and y variable it is oblique.

Recall in the previous section we graphed oblique lines that were given in the form $y = \underline{\quad} x + \underline{\quad}$ by making a table of values and graphing the

co-ordinate points. This form of the equation is called slope – intercept form. This will be expanded upon in grade 10 math.

Linear equations can be written in other forms also.

The form $__x + __y = __$ where the $__$ may be any number is called standard form.

Graphing Using Standard Form:

Method 1: Substitute values into the given equation and make a table of values.

Ex: Graph $2x - y = 8$ for $x = -2, 0$ and 2

To make a table of values, substitute each value of x in the equation and solve for y .

Substitute $x = -2$

$$2x - y = 8$$

$$2(-2) - y = 8$$

$$-4 - y = 8$$

$$\cancel{-4} + \cancel{4} - y = 8 + 4$$

$$-y = 12$$

$$\underline{-y = 12}$$

$$-1 \quad -1$$

$$y = -12$$

Substitute $x = 0$

$$2x - y = 8$$

$$2(0) - y = 8$$

$$0 - y = 8$$

$$-y = 8$$

$$\underline{-y = 8}$$

$$-1 \quad -1$$

$$y = -8$$

Substitute $x = 2$

$$2x - y = 8$$

$$2(2) - y = 8$$

$$4 - y = 8$$

$$\cancel{4} \cancel{-4} - y = 8 - 4$$

$$-y = 4$$

$$\underline{-y = 4}$$

$$-1 \quad -1$$

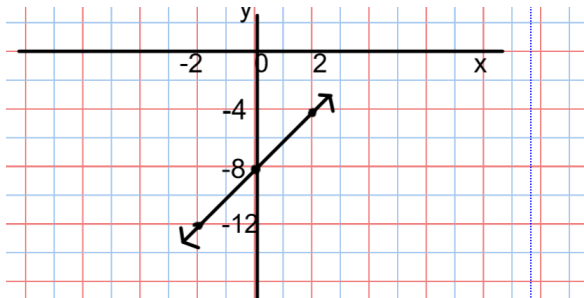
$$y = -4$$

The table of values would look like

x	y
-2	-12
0	-8
2	-4

Plot the points and join them.

The graph would be



Method 2: Rearrange the equation and then substitute values in to the rearranged equation.

Ex 2: Graph $3x - 2y = 6$ for $x = -4, 0$ and 4

Rearrange $3x - 2y = 6$ so that y is on one side of the equation by itself

$$3x - 2y = 6$$

$$\cancel{3x} - \cancel{3x} - 2y = 6 - 3x$$

$$-2y = 6 - 3x$$

$$\frac{-2y}{-2} = \frac{6}{-2} - \frac{3x}{-2}$$

$$y = -3 + \frac{3}{2}x$$

Use the rearranged equation and substitute values of x in the equation and solve for y

Substitute $x = -4$

$$y = -3 + \frac{3}{2}x$$

$$y = -3 + \frac{3}{2}(-4)$$

$$y = -3 + -6$$

$$y = -9$$

Substitute $x = 0$

$$y = -3 + \frac{3}{2}x$$

$$y = -3 + \frac{3}{2}(0)$$

$$y = -3$$

Substitute $x = 4$

$$y = -3 + \frac{3}{2}x$$

$$y = -3 + \frac{3}{2}(4)$$

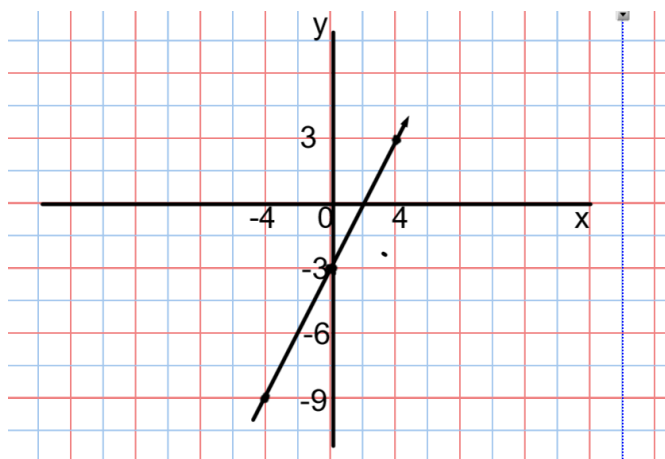
$$y = -3 + 6$$

$$y = 3$$

So the table of values would be

x	y
-4	-9
0	-3
4	3

And the graph



You may use either method to graph oblique lines but all work must be shown.

Problem Solving:

Ex: Two integers have a sum of 10.

(a) Make a table of values with at least 6 possible pairs of integers.

(b) Graph the relation between the two integers.

(c) Write an equation to represent the relation between the integers

Solution:

(a) Let x and y represent the two integers

x	y
-4	14
-2	12
0	10
2	8
4	6
6	4

Note: The question asks for integers so negative values are included in the table but not fractions or decimals. Points on the graph will not be connected.



(c) The equation would be

$$x + y = 10$$



Do Practice Questions p.178-180 #4 to 18

Do Section 4.3 Quiz

Focus 4: Match equations and graphs of linear relations

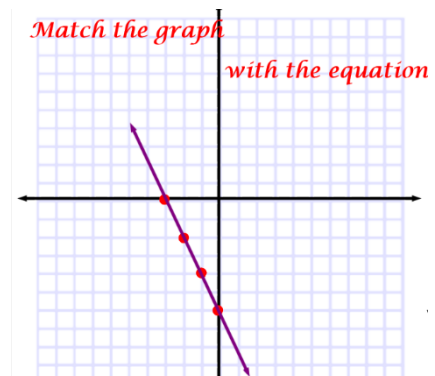
Method 1: Using a table of Values

In the previous sections you have drawn graphs from an equation using a table of values. In this section the graph is drawn for you. By making a table of values, you can determine which equation matches a given graph.

Example 1:

Which of the following equations matches this graph?

- (a) $y = 2x - 6$
- (b) $y = -2x + 6$
- (c) $y = -2x - 6$



Solution: For each equation , make a table of values

(a) $y = 2x - 6$

x	y
-2	-10
0	-6
2	-2

(b) $y = -2x + 6$

x	y
-2	10
0	6
2	2

(c) $y = -2x - 6$

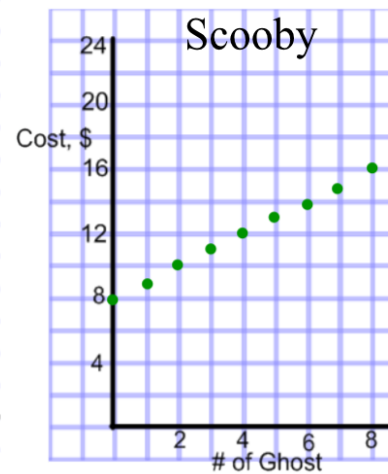
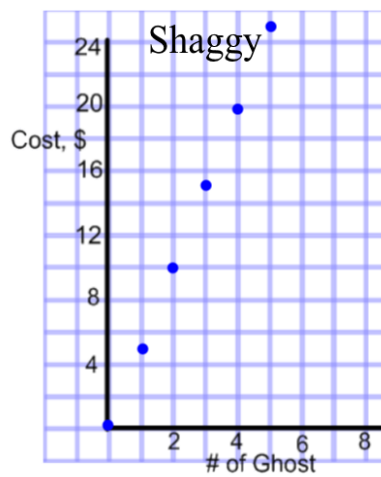
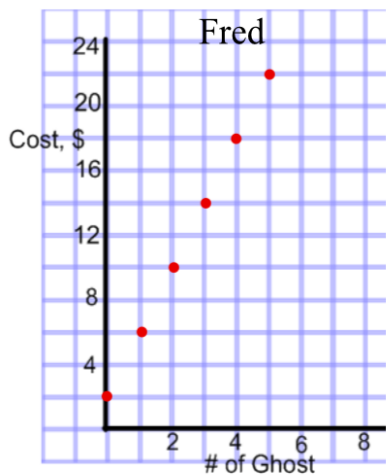
x	y
-2	-2
0	-6
2	-10

The table containing the set of co-ordinate points on the graph is c , so the equation that matches the graph is $y = -2x - 6$.

TRY THIS



Fred, Shaggy and Scooby are hired to find ghosts. Each ghost hunter charges a different rate. These graphs show how the cost is related to the number of ghosts caught.



Match each graph with its equation:

$y = x + 8$

$y = 5x$

$y = 4x + 3$

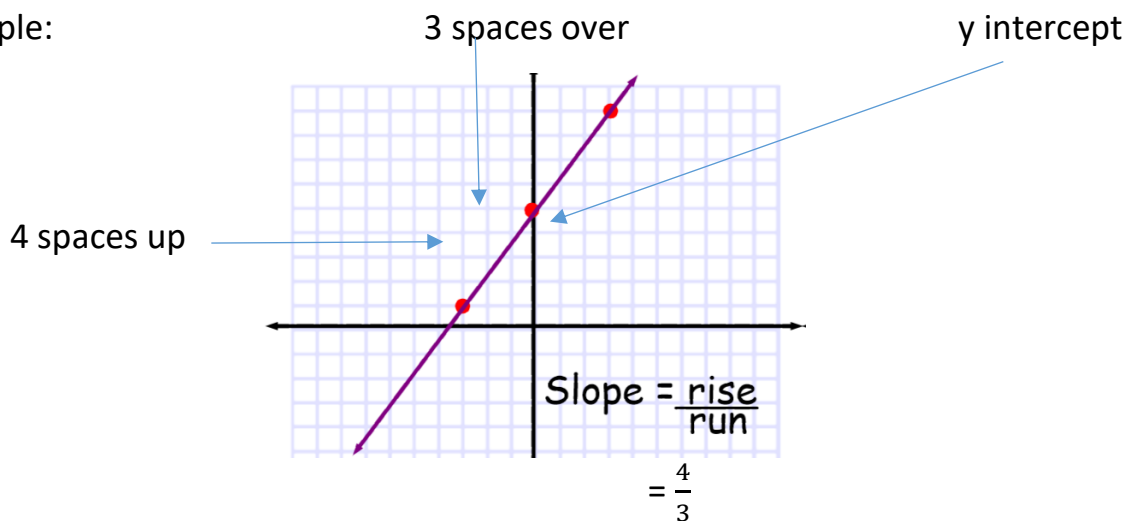
ENRICHMENT

Method 2: Using the y intercept and the slope of the line.

The slope of a line can be found by picking 2 points on the line and using the leftmost point count how far up or down you must go to get to the second point. This is the rise. The run is calculated by taking the leftmost point and counting how far to the right you must go to get to the second point. Then

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

Example:



To find the equation of the line also identify the y intercept (where the graph crosses the y axis) . In this case the y intercept is 5.

The equation of the line can be written in slope intercept form as

$$y = mx + b$$

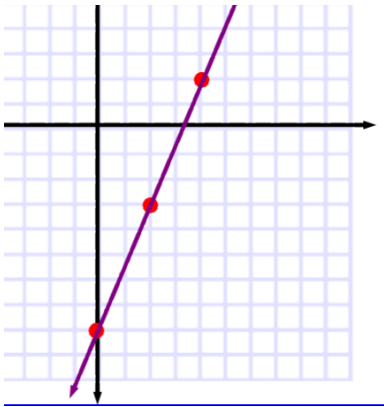
where m is the slope and b is the y intercept

So the equation for this line is $y = \frac{4}{3}x + 5$

Slope $\frac{4}{3}$ y intercept 5

TRY THIS

Find the equation of the line below in slope intercept form.



This method can also be used to match a graph with its equation.

Example 2:

The 3 graphs below have these equations, but the graphs are not in order:

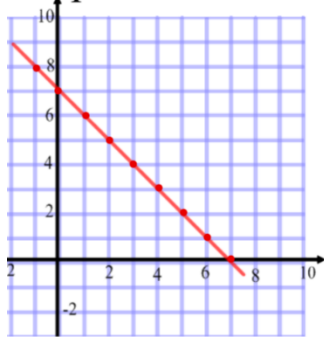
$$y = 2x + 4$$

$$x + y = 7$$

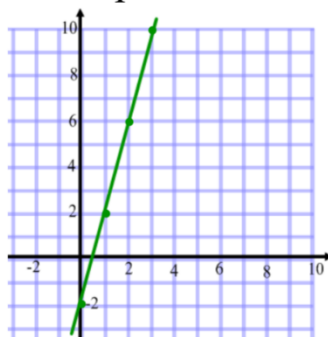
$$y = 4x - 2$$



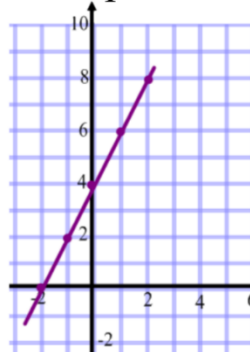
Graph A



Graph B



Graph C



Match the graphs.

Find the slopes and y intercepts from each of the equations. Note they must be in the form $y = mx + b$

(1) $y = 2x + 4$ has a slope of 2 and a y intercept of 4. The only graph that has a y intercept of 4 is graph C. Using 2 points on the graph it can be shown that the slope of the graph is 2. So $y = 2x + 4$ matches with C.

(2) $x + y = 7$. This equation is not in slope intercept form so it must be changed to this form before the slope and intercept can be identified from the equation

Rearrange the equation for y

$$x + y = 7$$

$$\cancel{x} - \cancel{x} + y = 7 - \cancel{x}$$

$$y = 7 - x \quad \text{or} \quad y = -x + 7$$

The slope of the line is -1 and the y intercept is 7.

The equation with an intercept of 7 and a slope of -1 is A . So the equation for A is $x + y = 7$

(3) $y = 4x - 2$ has a y intercept of -2 and a slope of 4. Graph B crosses the y axis at -2 and has a slope of 4 so Graph B has the equation $y = 4x - 2$



Do practice questions p.188-190 #3 to 13

Do BLM 4.4 and pass in to get marked.

Focus 5: Use Interpolation and extrapolation to estimate values on a graph.

A cup is placed under a leaking tap. The following graph shows the relationship between the amount of water in the cup in milliliters(mL) as a function of the amount of time that has passed in seconds(s).

Water in a Cup



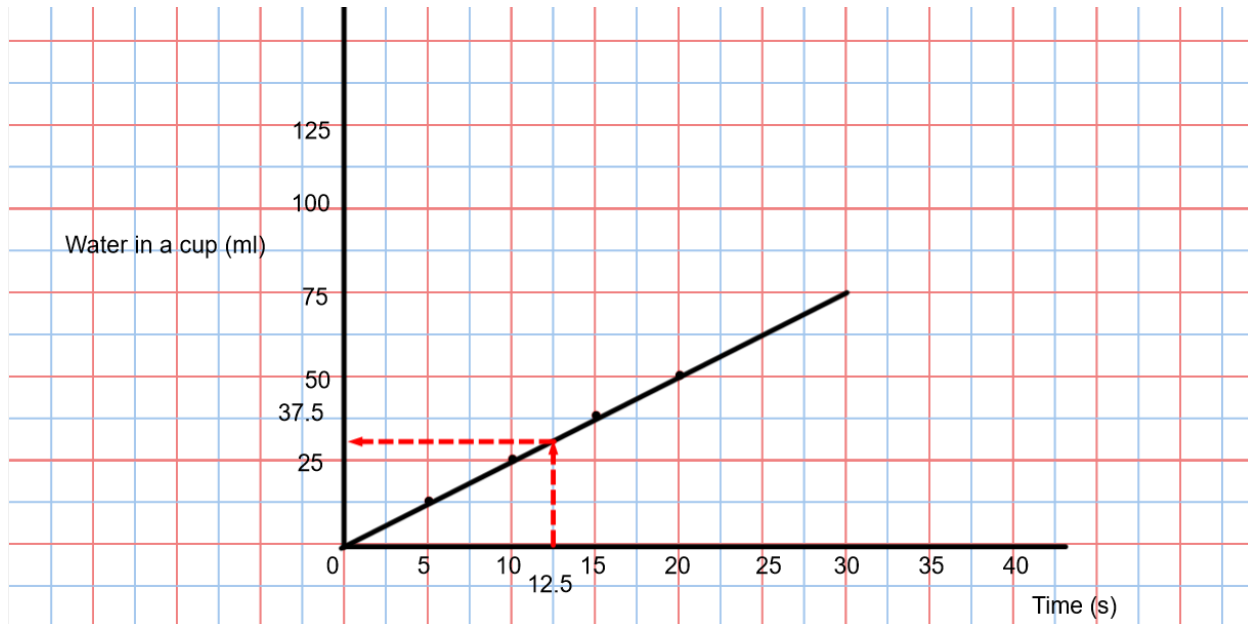
We can use the graph to estimate values that lie between two data points on the graph. This is called **interpolation**.

For example to estimate how much water is in the cup after 12.5 s

- We would begin at 12.5 s on the time axis
- Draw a vertical line to the graph
- Then draw a horizontal line from the graph to the axis with the amount of water
- The line intersects the axis at **approximately 32 ml**.
So there would be about 32 ml of water in the cup after 12.5 s.

SEE GRAPH ON NEXT PAGE

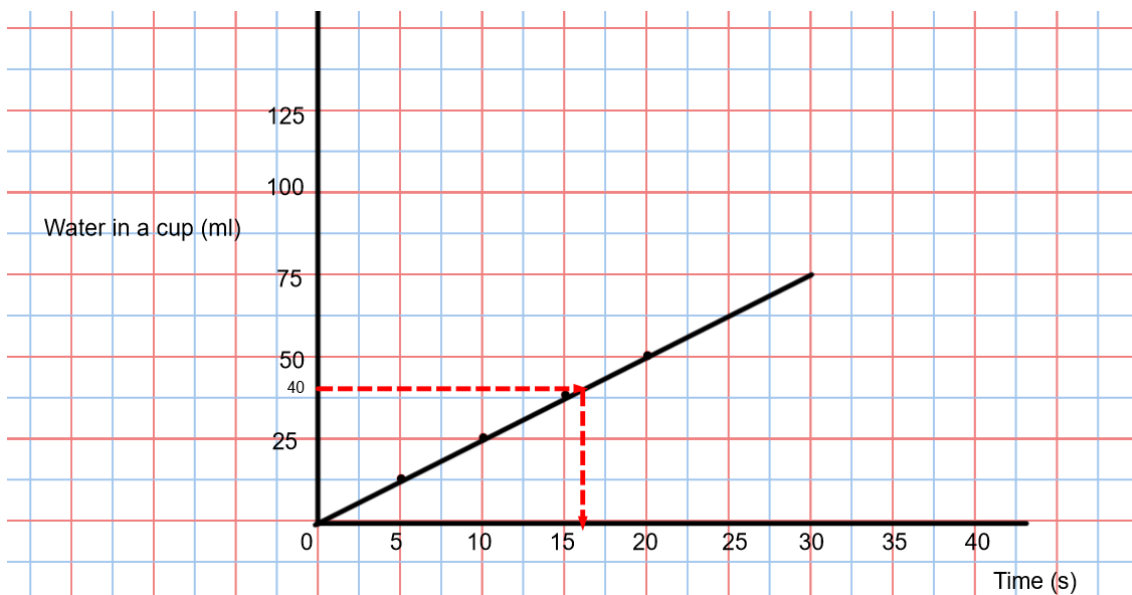
Water in a Cup



To estimate the time it takes for 40 mL of water to drip into the cup

- Begin at 40 mL on the vertical axis
- Draw a horizontal line to the graph
- Draw a vertical line to the time axis
- The value at this point on the axis is approximately 16s.

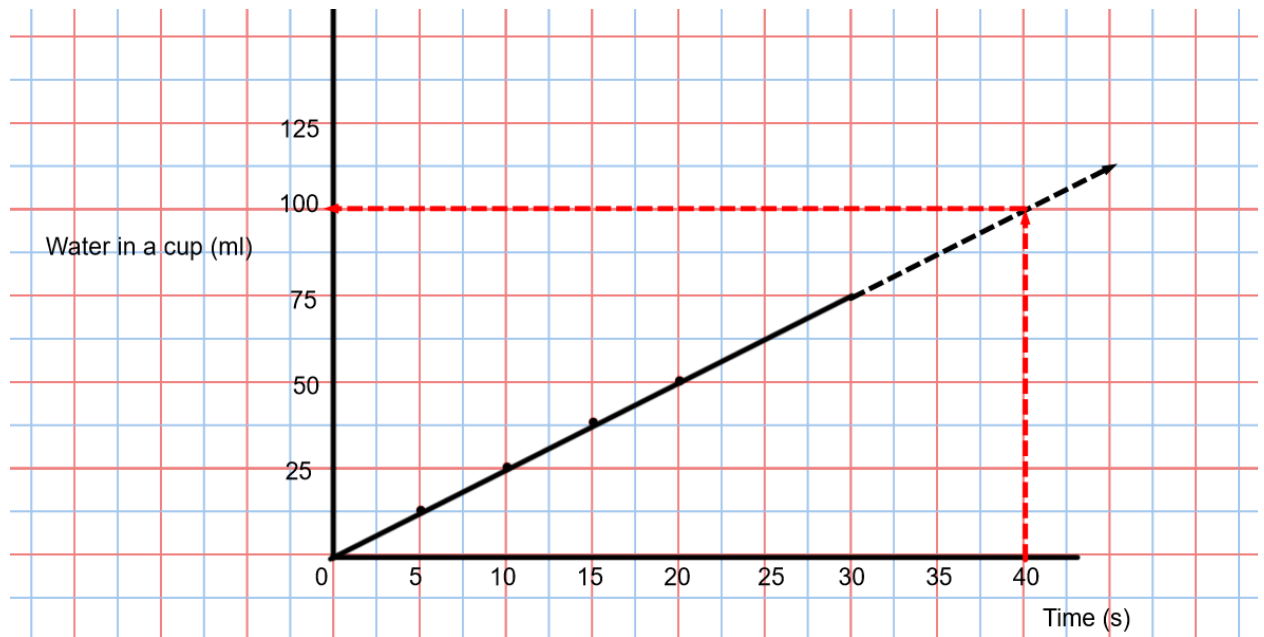
It takes about 16s for 40mL of water to leak from the tap into the cup.



Suppose the tap continues to leak at the same rate. We can extend the graph to predict how much water will be in the cup after 40s. This is called **extrapolation**.

- Extend the graph (using a ruler!!) until the time axis shows 40s.
- Repeat the same process as you followed for 12.5s.

Water in the Cup

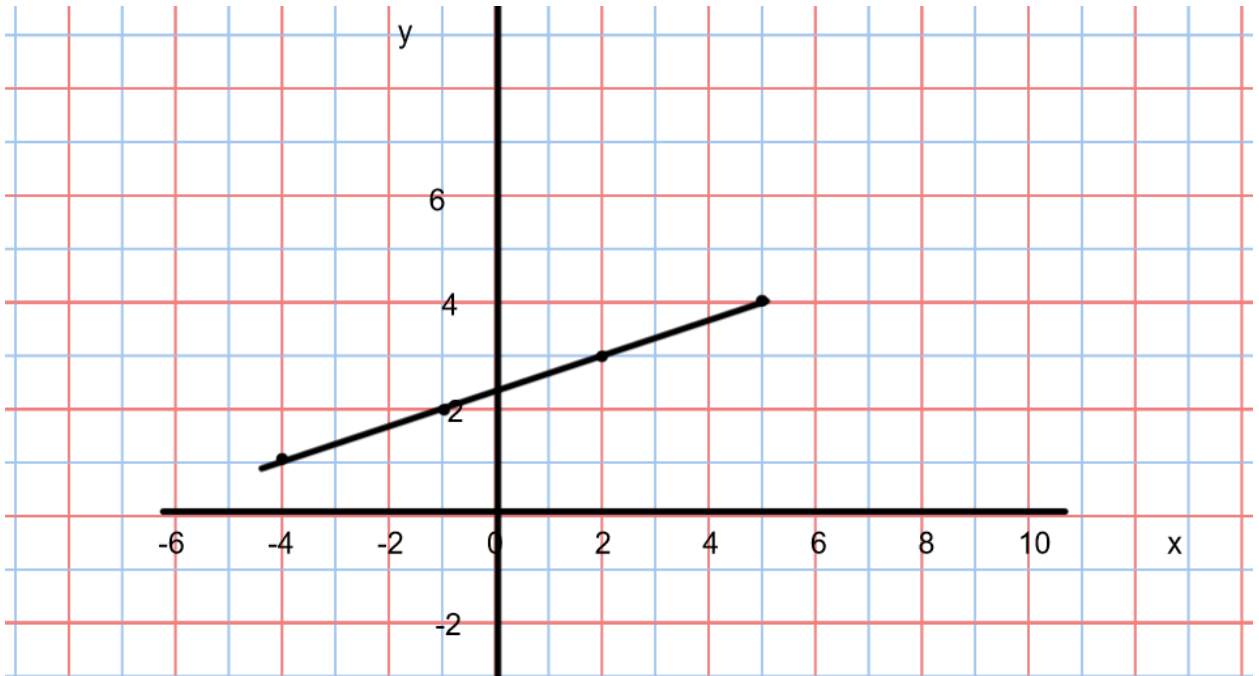


After 40 s, there would be approximately 100 mL of water in the cup.

Example 1: Interpolating and Extrapolating to Determine Values of Variables From a Graph

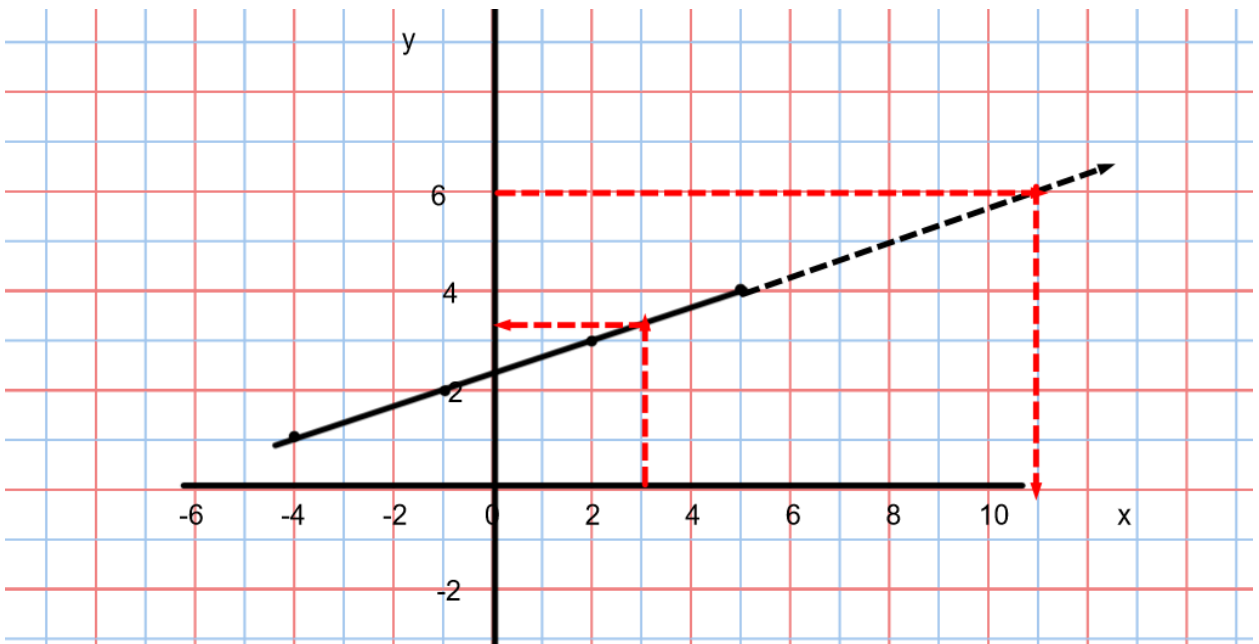
Using the graph that follows

- Determine the value of x when $y = 6$
- Determine the value of y when $x = 3$



Solution :

(a) Extend the graph to the right to **extrapolate** for $y = 6$



Begin at 6 on the y axis. Move across the graph, then down to the x axis.
When $y = 6$, $x = 11$

(b) **Interpolate** for $x = 3$.

Begin at 3 on the x axis. Move up to the graph, then across to the y axis.

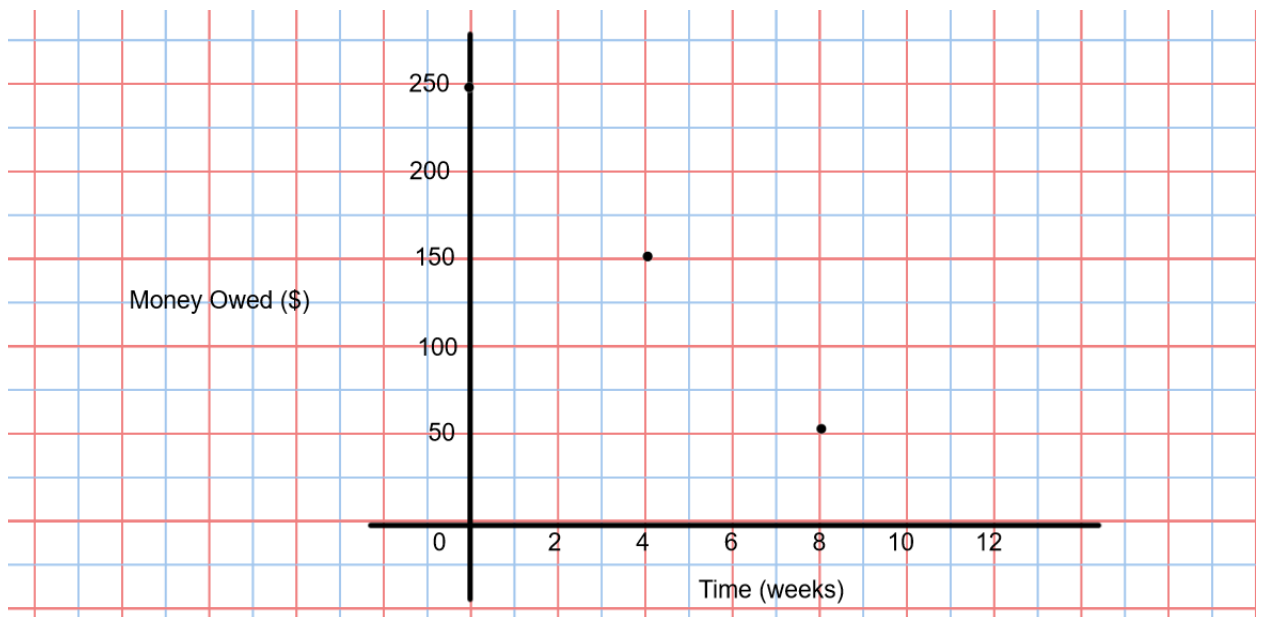
The value of y is between 3 and 4, but closer to 3.

When $x = 3$, $y = 3\frac{1}{3}$

Example 2: Using Interpolation and Extrapolation to Solve Problems.

Joe borrows money from his parents to buy a new phone. He repays the loan by making regular weekly payments. The graph shows how the money is repaid over time. The data are discrete because payments are made just once a week.

Joe's Loan Payments

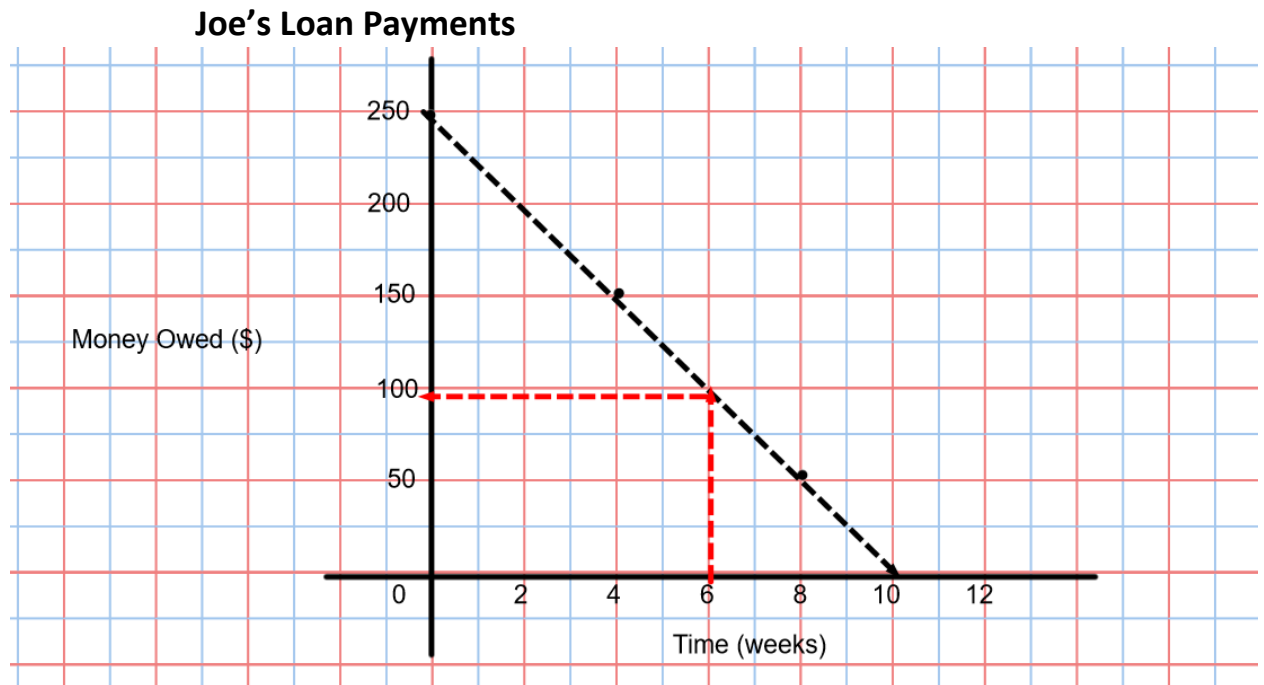


(a) How much did Joe originally borrow?

(b) How much does Joe owe after 6 weeks?

(c) How long will it take Joe to pay the loan off?

Solution: Even though the data are discrete it is helpful to join the points on the graph in order to interpolate or extrapolate values from the graph.



(a) At time 0 the money is borrowed. This is the point where the graph crosses the vertical axis. Joe borrowed \$250.

(b) Begin at 6 on the time axis. Draw a vertical line to the graph then a horizontal line to the Money Owed axis. The amount owed after 6 weeks is \$100.

(c) When all the money has been repaid the money owing will be 0. This is the point on the time axis where the graph crosses it. After 10 weeks the loan will be repaid.



Complete Practice Questions p. 196-198 #4 to 15
Complete BLM 4.5 and pass in to be corrected

Review for Test p.201-203 #1 to 17
Do Unit 7 Test