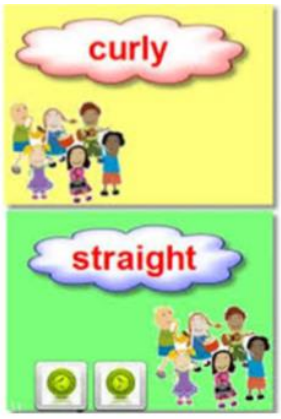
Math 9

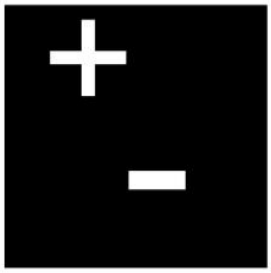
Module 6

PR3: Model and solve problems using linear equations of the form:
$ax = b$; = b , a $\neq$ 0; $ax + b = c$; $+b = c$, a $\neq$ 0; = b, $x \neq 0$ ax ax xa
$ax + b = cx + d$; $a(bx + c) = d(ex + f)$; $a(x + b) = c$; ax = b+cx
concretely, pictorially and symbolically, where a, b, c, d, e, *and* f are rational numbers.

PR4: Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.

Focus 1: Solving Problems by Using Inverse Operations



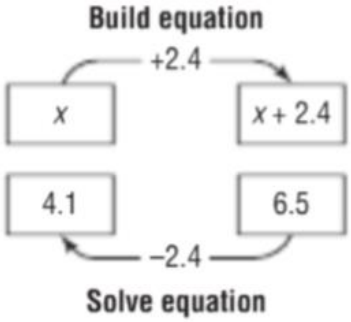


Inverse operations "undo" or reverse each other's results.
Addition and subtraction are inverse operations.
Multiplication and division are also inverse operations.

We can use inverse operations to solve many types of equations. To do this, we determine the operations that were applied to the variable to build the equation. We then use inverse operations to isolate the variable by "undoing" these operations.

For example, to solve $x + 2.4 = 6.5$:

- Start with x.
  Identify the operation applied to x to produce the expression $x + 2.4$; that is, add 2.4 to get:
  $x + 2.4$
- Since $x + 2.4$ is equal to 6.5, apply the inverse operation on 6.5 to isolate x; that is, subtract 2.4 to get:

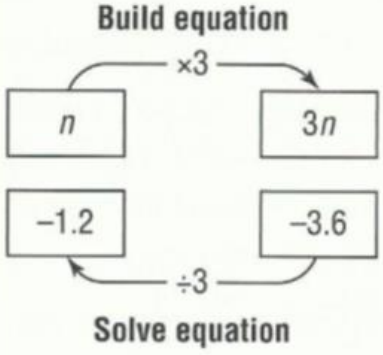


Solutions:

a) Let n represent the number. Then 3 times n is -3.6.

The equation is : 3n = -3.6

Inverse Operations



Algebraic Solution

$3n = -3.6$

Undo the multiplication.

Divide each side by 3.

$\frac{3n}{3} = \frac{-3.6}{3}$

$n = -1.2$

Verify the solution: 3 (-1.2) = -3.6 so the solution is correct

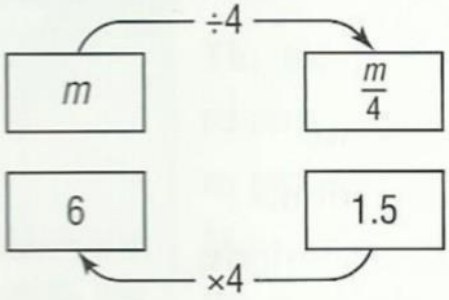
b) A number divided by 4 is 1.5

b) Let m represent the number. Then, m divided by 4 is 1.5.

The equation is: $\frac{m}{4} = 1.5$

Inverse Operations

Build equation



Solve equation

Algebraic Solution

$\frac{m}{4} = 1.5$

Undo the division.

Multiply each side by 4.

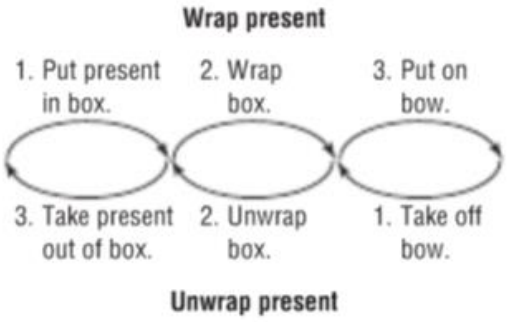
$4 \times \frac{m}{4} = 4 \times 1.5$

$m = 6$

Verify the solution $\frac{6}{4} = 1.5$, so the solution is correct.

Solving a Two-Step Equation

To "undo" a sequence of operations, we perform the inverse operations in the reverse order. For example, compare the steps and operations to wrap a present with the steps and operations to unwrap the present.



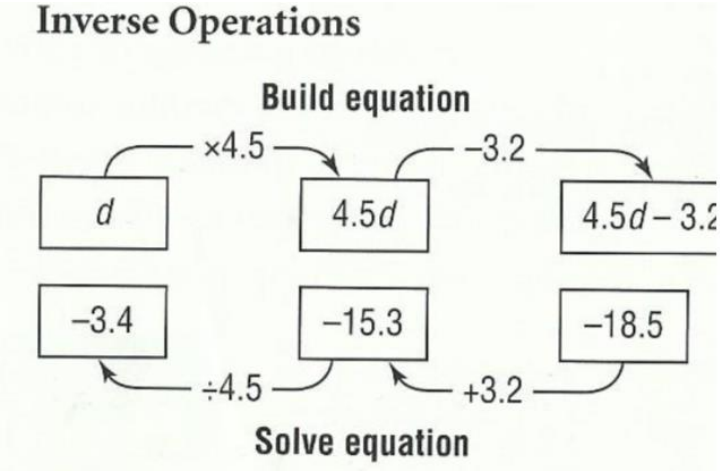
Give it a try using inverse operations and algebraic solution.

a) $4.5d - 3.2 = -18.5$

b) $\frac{r}{4} + 3 = 7.2$

SOLUTIONS

a) 4.5 d – 3.2 = -18.5



left side

right side

OR

Solve

$4.5d - 3.2 + 3.2 = -18.5 + 3.2$ (add 3.2 to both sides)

$4.5d = -15.3$

$\frac{4.5d}{4.5} = \frac{-15.3}{4.5}$ (divide each side by 4.5)

$d = -3.4$

Verify

If d = -3.4

$4.5d - 3.2 = -18.5$

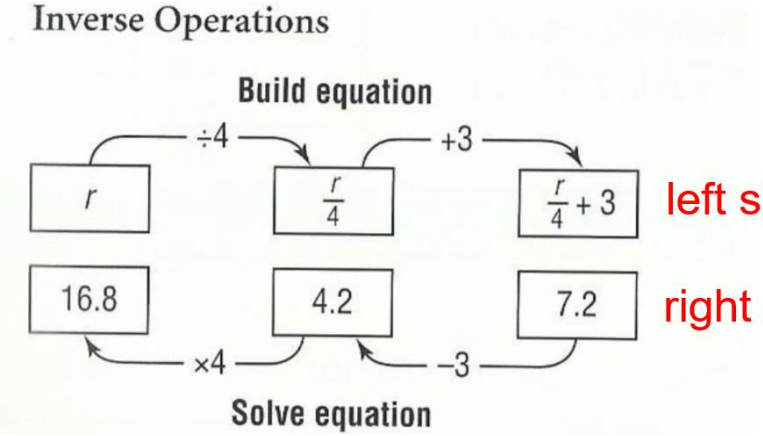
$4.5(-3.4) - 3.2 = -18.5$

$-15.3 - 3.2 = -18.5$

$-18.5 = -18.5$

Left side equals the right side so solution is correct.

b.) $\frac{r}{4} + 3 = 7.2$



left side

right side

OR

Solve

$\frac{r}{4} + 3 = 7.2$

$\frac{r}{4} + 3 - 3 = 7.2 - 3$ (subtract 3 from each side)

$\frac{r}{4} = 4.2$ (multiply each side by 4)

Verify

Note: You must write

If r = 16.8

$\frac{r}{4} + 3 = 7.2$

$\frac{16.8}{4} + 3 = 7.2$

$4.2 + 3 = 7.2$

$7.2 = 7.2$

Try question #'s 5-9 on page 271 in your text

Example 3 Using an Equation to Model and Solve a Problem

A rectangle has length 3.7 cm and perimeter 13.2 cm.

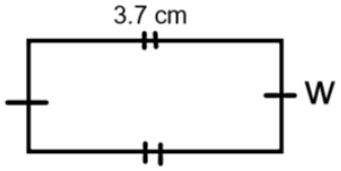
a) Write an equation that can be used to determine the width of the rectangle.

b) Solve the equation.

c) Verify the solution.

Solution:

We know the perimeter of a rectangle is twice the sum of the length and width.



a.) Let w represent the wide of the rectangle.

So, 13.2 = 2 (3.7 + w)

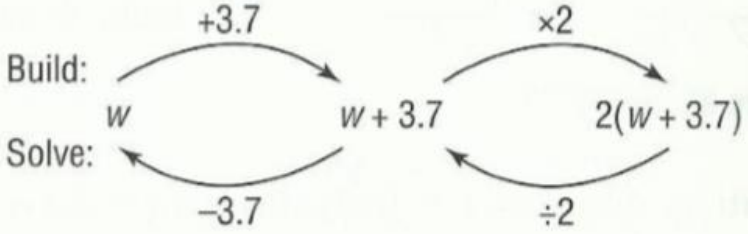
Solve the equation

Method 1

Use inverse operations.

$13.2 = 2(3.7 + w)$

Think:



$$13.2 = 2(3.7 + w)$$ Divide each side by 2.

$$\frac{13.2}{2} = \frac{2(3.7 + w)}{2}$$

$$6.6 = 3.7 + w$$ Subtract 3.7 from each side.

$$6.6 - 3.7 = 3.7 + w - 3.7$$

$$2.9 = w$$

Method 2

Use the distributive property, then inverse operations.

$13.2 = 2(3.7 + w)$ Use the distributive property to expand 2(3.7 + w)

$13.2 = 2(3.7) + 2(w)$

$13.2 = 7.4 + 2w$

$13.2 - 7.4 = 7.4 + 2w - 7.4$ Subtract 7.4 from each side.

$5.8 = 2w$ Divide each side by 2.

$\frac{5.8}{2} = \frac{2w}{2}$

$2.9 = w$

c) Check: The perimeter of a rectangle with length 3.7 cm and width 2.9 cm is:

$2(3.7 \text{ cm} + 2.9 \text{ cm}) = 2(6.6 \text{ cm})$

$= 13.2 \text{ cm}$

The solution is correct. The width of the rectangle is 2.9 cm.

Example 4 Using an Equation to Solve a Percent Problem

Seven percent of a number is 56.7.

a) Write, then solve an equation to determine the number.

b) Check the solution.

Give it a try

Example 4 Using an Equation to Solve a Percent Problem

Seven percent of a number is 56.7.

a) Write, then solve an equation to determine the number.

b) Check the solution.

A Solution

a) Let n represent the number. Then, 7% of the number is $7\% \times n$, or $0.07n$.

An equation is: $0.07n = 56.7$

$0.07n = 56.7$ Divide each side by 0.07.

$\frac{0.07n}{0.07} = \frac{56.7}{0.07}$ Use a calculator.

$n = 810$

The number is 810.

b) 7% of $810 = 0.07 \times 810$

$= 56.7$

Try question #'s 10-23 on page 273 - 274 in your text

Do BLM 6.18 & pass in

** Quiz Focus 1**

Focus 2: Solving Equations by Using Balance Strategies

- **The two sides of the equation must always be balanced. To keep it balanced you do the same operation on both sides of the equation**
- **You rearrange the equation so all terms containing the variable are on the same side of the equation.**
- **Then isolate the variable to solve**

Type 1 – Variables on both sides of the equation

$-3c + 7 = 2c - 8$

$-2c - 3c + 7 = \cancel{-2c + 2c} - 8$ Subtract 2c from both sides of the equation

$-5c + 7 = -8$ To isolate the variable you must subtract 7

$-5c + \cancel{7 - 7} = -8 - 7$ from both sides of the equation

$-5c = -15$

$\frac{-5c}{-5} = \frac{-15}{-5}$ Then divide both sides of the equation by -5

$c = 3$

Verify	**Left Side**	**Right Side**
	$-3c + 7$	$2c - 8$
	$-3(3) + 7$	$2(3) - 8$
	$-9 + 7$	$6 - 8$
	-2	-2
	✓	✓

- **Try this one.** **$-2.4a + 3.7 = -16.1 + 3.1a$** **Verify your solution**

Solution: $a = 3.6$

Type 2 – Equations with fraction (s)

Example 1 - Variable is in the numerator

$$\frac{r}{4} + 3 = 12$$

$\frac{r}{4} + 3 - 3 = 12 - 3$ Subtract 3 from each side

$\frac{r}{4} = 9$ You get

$4 \text{ x } \frac{r}{4} = 9 \text{ x } 4$ Multiply each side by 4, because multiplying is the inverse operation of dividing

r = 36

Verify your solution **Left Side** **Right Side**

$\frac{r}{4} + 3$ 12

$\frac{36}{4} + 3$

9 + 3

12

✓ ✓

- **Try this one:** $\frac{-w}{10} + 6 = 12$ **Verify - ALL WORK MUST BE SHOWN Solution w = -60**

Example 2 – Variable is in the denominator

$$\frac{18}{s} + 3 = 12$$

$\frac{18}{s} + 3 - 3 = 12 - 3$ Subtract 3 from each side

$\frac{18}{s} = 9$ You get

$s \text{ x } \frac{18}{s} = 9 \text{ x } s$ Multiply each side by s, because multiplying is the inverse operation of dividing

18 = 9s

$\frac{18}{9} = \frac{9s}{9}$

2 = x

Verify your solution

Left Side	**Right Side**
$\frac{18}{s} + 3$	12
$\frac{18}{2} + 3$	
9 + 3	
12	
✓	✓

Type 3 Equations with multiple fractions

$$\frac{4a}{5} = \frac{2a}{3} + 2$$

$15\left(\frac{4a}{5}\right) = 15\left(\frac{2a}{3}\right) + 15(2)$ To get rid of the fraction, multiply all terms by the common denominator 15

$^{3}\cancel{15}\left(\frac{4a}{\cancel{5}}\right) = {}^{3}\cancel{15}\left(\frac{2a}{\cancel{3}}\right) + 15(2)$

$3(4a) = 3(2a) + 15(2)$ ** This line is does not need to be shown

$12a = 6a + 30$ Subtract 6a from each side

$-6a + 12a = -6a + 6a + 30$

$6a = 30$

$\frac{6a}{6} = \frac{30}{6}$ Divide each side of the equation by 6

a = 5

Type 3 – Equations with Brackets

********Distributive Property********

Multiply everything inside the bracket by the number in front

$3(a + 5) = 4(a - 3)$

$3a + 15 = 4a - 12$

Example

$2.2(h - 5.3) = 0.2(-32.9 + h)$

$2.2h - 11.66 = -6.58 + 0.2h$ Distributive Property on both sides of the equation.

$-0.2h + 2.2h - 11.66 = -6.58 + \cancel{0.2h} - \cancel{0.2h}$ Subtract -0.2h on both sides. All variables will now be on the left side of the equation

$2h - 11.66 + 11.66 = -6.58 + 11.66$ Move all numbers to the right-hand side of the equation

$\frac{2h}{2} = \frac{5.08}{2}$ Divide both sides of the equation by 2

$h = 2.54$ Solution

Note: Your answer can have a decimal because the question had decimals

VERIFY

Left Side	**Right Side**	
$2.2(h - 5.3)$	$0.2(-32.9 + h)$	Substitute 2.54 where h is
$2.2(2.54 - 5.3)$	$0.2(-32.9 + 2.54)$	
$2.2(-2.76)$	$0.2(-30.36)$	
-6.072	-6.072	
✓	✓	

Try this one: $3(4j + 5) = 2(-10 + 5j)$ Verify **ALL WORK MUST BE SHOWN**

Solution j= -17.5

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Questions 15 -21

*** Mid Unit Quiz Focus 2

FOCUS 3 – Solving Linear Inequalities by using Addition and Subtraction

***** To solve linear inequalities by addition or subtraction use the same rules used for solving an equation**

- **What you do to one side you must do to the other side**

Example 1 **Solving an Inequality**

a) Solve the inequality: $6.2 \leq x - 4.5$ b) Verify the solution. c) Graph the solution.

a) $6.2 \leq x - 4.5$ Isoloate the variable (get the term with a variable on one side of the equation)

$4.5 + 6.2 \leq x - 4.5 + 4.5$ Add 4.5 to both sides of the equation

$10.7 \leq x$

Or

$X \geq 10.7$

b) Verify the solution

This means x is equal 10.7 or any number that is greater than 10.7.

To verify pick any number greater than 10.7 or equal to 10.7 and fill in the original inequality.

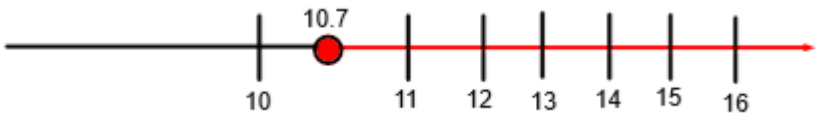
For example if we choose 12.

$6.2 \leq x - 4.5$

$6.2 \leq 12 - 4.5$

$6.2 \leq 7.5$ TRUE

c) Graph



Example 2 **Using an Inequality to Model and Solve a Problem**

Jake plans to board his dog while he is away on vacation.

- Boarding house A charges $90 plus $5 per day.
- Boarding house B charges $100 plus $4 per day.

For how many days must Jake board his dog for boarding house A to be less expensive than boarding house B?

a) Choose a variable and write an inequality that can be used to solve this problem.

b) Solve the problem.

c) Graph the solution.

a) Let d represent the number of days Jake will board his dog.
Boarding House A can be written as $90 + $5d
Boarding House B can be written as $100 + $4d
For Boarding House to be less expensive the inequality is

$$90 + 5d < 100 + 4d$$

b) Solve

$$90 + 5d < 100 + 4d$$

$$90 + 5d - 4d < 100 + 4d - 4d \qquad \text{Subtract 4d from each side}$$

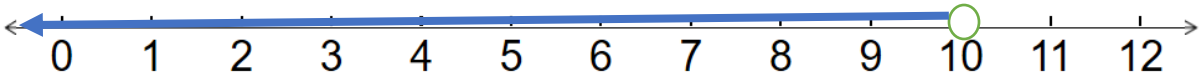
$$90 + d < 100$$

$$-90 + 90 + d < 100 - 90 \qquad \text{Subtract 90 from each side}$$

$$d < 10$$

Boarding house A is less expensive when Jake leaves his dog there for less than 10 days.

c) $d < 10$



Page 298-299 Questions 4 to 14

FOCUS 4 – Solving Linear Inequalities by using Multiplication and Division

*** When solving an inequality using multiplication or division all steps are the same as used to solve equations **except** when it comes to isolating….

Let's investigate, consider the inequality $-1 < 2$

Multiply each side by 3

$-1 < 2$

$(-1)(3) < (2)(3)$

$-3 < 6$

Divide each side by 3

$-1 < 2$

$(-1) \div (3) < (2) \div (3)$

$-\frac{1}{3} < \frac{2}{3}$

Multiply each side by -3

$-1 < 2$

$(-1)(-3) < (2)(-3)$

$3 > -6$

Divide each side by -3

$-1 < 2$

$(-1) \div (-3) < (2) \div (-3)$

$\frac{1}{3} > -\frac{2}{3}$

You must reverse the inequality sign for each inequality to remain true.

The examples above illustrate the properties of inequalities

- When each side of an inequality is multiplied or divided by the same positive number, the resulting inequality is true.
- When each side of an inequality is multiplied or divided by the same negative number, the inequality sign must be reversed for the inequality to remain true.

RULE: If you multiply or divide an inequality by a negative, reverse the inequality sign!!!

Example 1 – Solving a one-step inequality

a) **-5s ≤ 25**

$\frac{-5S}{-5} \geq \frac{25}{-5}$ ****Reverse the sign as you divide by -5 for the inequality to remain true

S ≥ -5

b) **7a ≤ 21**

$\frac{7a}{7} \leq \frac{21}{7}$

a ≤ 3

c) $\frac{\mathbf{y}}{\mathbf{-4}} > \mathbf{-3}$

$\mathbf{-4}\left(\frac{y}{-4}\right) < \mathbf{-4}(-3)$ ****Reverse the sign as you multiply by -4 for the inequality to remain true

y < 12

d) $\frac{\boldsymbol{k}}{\mathbf{3}} \geq$ **-2**

$3\left(\frac{k}{3}\right) \geq 3(-2)$

$k \geq -6$

Example 1 – Solving a multi-step inequality

a) Solve the inequality –2.6a + 14.6 > -5.2 + 1.8a
b) Verify the solution

a) -2.6a + 14.6 > -5.2 + 1.8a

-2.6a + 14.6 -14.6 > -14.6 -5.2 + 1.8a **Subtract 14.6 from each side**

-2.6a > -19.8 + 1.8a

- 1.8a – 2.6a > -19.8 +1.8a – 1.8a **Subtract 1.8a from each side**

- 4.4a > -19.8

$\frac{-4.4a}{-4.4} < \frac{-19.8}{-4.4}$ **Divide both side by -4.4**

reverse the inequality sign

a < 4.5

b) The solution of the inequality a<4.5 is all numbers less than 4.5
-Choose several numbers less than 4.5, for example 4, 0 and - 2
-Substitute into the original equation

a= 4

Left Side		Right Side
- 2.6a + 14.6	>	-5.2 + 1.8a
-2.6(4) + 14.6		-5.2 + 1.8 (4)
-10.4 + 14.6		-5.2 + 7.2
4.2		2

Since 4.2 > 2, a=4 satisfies the inequality

a =0

Left Side		Right Side
- 2.6a + 14.6	>	-5.2 + 1.8a
-2.6(0) + 14.6		-5.2 + 1.8(0)
14.6		-5.2

Since 14.6 > -5.2, a=0 satisfies the equation

a= 2

Left Side		Right Side
- 2.6a + 14.6	>	-5.2 + 1.8a
-2.6(-2) + 14.6		-5.2 + 1.8 (-2)
5.2 + 14.6		-5.2 + -3.6
19.8		-8.8

Since 4.2 > 2, a=4 satisfies the inequality. Since all 3 substitutions verify the inequality, it suggests that a < 4.5 is correct

Example 3 **Using an Inequality to Model and Solve a Problem**

A super-slide charges $1.25 to rent a mat and $0.75 per ride. Haru has $10.25. How many rides can Haru go on?

a) Choose a variable, then write an inequality to solve this problem.
b) Solve the problem.
c) Graph the solution.

a) Let n represent the number of rides Haru can go on.
1.25 + 0.75n ≤ 10.25

b) **1.25 + 0.75n ≤ 10.25**

-1.25 + 1.25 + 0.75n ≤ 10.25 -1.25

0.75n ≤ 9

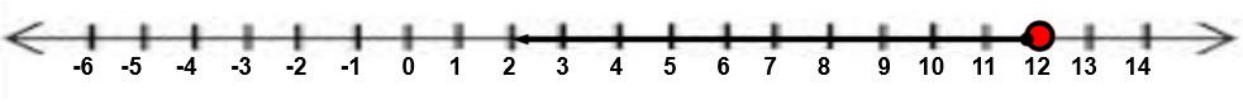
$$\frac{0.75n}{0.75} \leq \frac{9}{0.75}$$

$$n \leq 12$$

****Reminder – Word problems require a sentence**

Haru can go on 12 or fewer rides.

C)Graph



Do Page 305 -306 Questions 3 - 17

Do Review Page 308-309 Questions 3-16

Module 6 Test