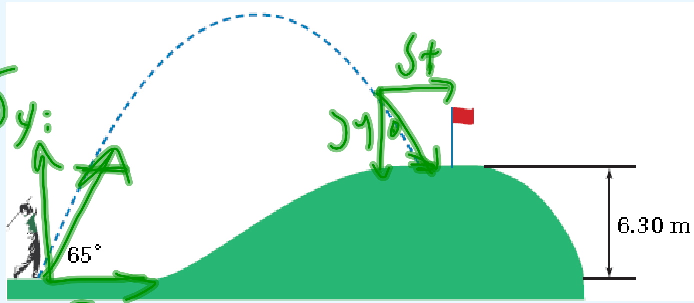
Projectiles Launched at an Angle

Analyzing Parabolic Trajectories

1. A golfer hits the golf ball off the tee, giving it an initial velocity of 32.6 m/s at an angle of 65° with the horizontal. The green where the golf ball lands is 6.30 m higher than the tee, as shown in the illustration. Find
   (a) the time interval during which the golf ball was in the air
   (b) the horizontal distance that it travelled
   (c) the velocity of the ball just before it hit the ground (neglect air friction)



δx

$V_x t = d_x$

Identify the Goal

(a) The time interval, Δt, that the golf ball was in the air

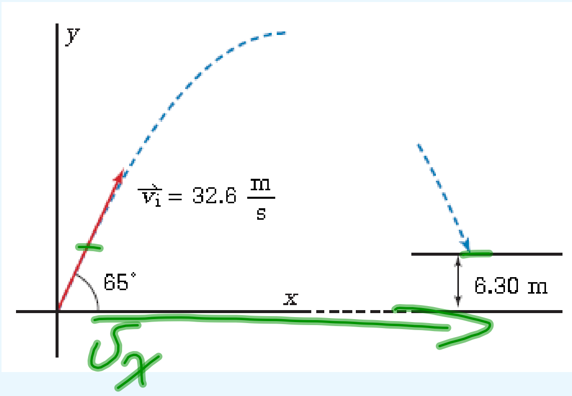
(b) The horizontal distance, Δx, that the golf ball travelled

(c) The final velocity of the golf ball, $\vec{v}_f$

Variables and Constants

Known		Implied	Unknown	
$\lvert\vec{v}_i\rvert = 32.6\ \frac{m}{s}$	$\Delta y = 6.30$ m	$a_y = -9.81\ \frac{m}{s^2}$	Δt	$\vec{v}_f$
$\theta_i = 65°$			Δx	θ_f
			v_{ix}	v_{iy}

Frame the Problem

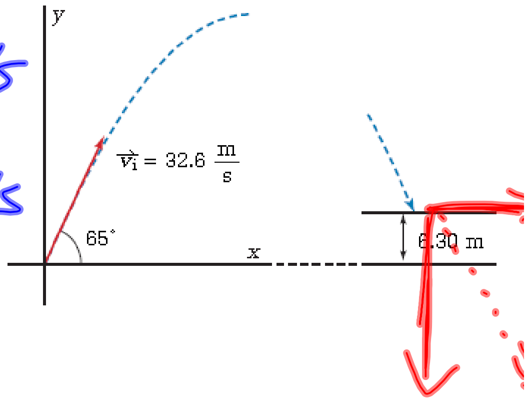


- Start to frame the problem by making a sketch that includes a *coordinate system*, the *initial conditions*, and all of the known information.
- The golf ball has a *positive initial velocity* in the *vertical* direction. It will rise and then fall according to the kinematic equations.
- The *vertical acceleration* of the golf ball is *negative* and has the *magnitude* of the acceleration due to *gravity*.
- The *time interval* is determined by the *vertical* motion. The *time interval ends* when the golf ball is at a height equal to the *height of the green*.
- The golf ball will be at the height of the green *twice*, once while it is *rising* and once while it is *falling*.
- Motion in the *horizontal* direction is *uniform*; that is, it has a *constant velocity*.
- The horizontal *displacement of the ball* depends on the *horizontal component* of the *initial velocity* and on the duration of the flight.

Calculations

$V_{ix} = 32.6 \cos 65 = 13.8 \text{ m/s}$

$V_{iy} = 32.6 \sin 65 = 29.5 \text{ m/s}$



$d = \frac{1}{2}at^2 + V_i t$

$6.3 = \frac{-9.8}{2}t^2 + 29.5t$

$-4.9t^2 + 29.5t - 6.3 = 0$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$t = 0.22\text{ s}$, 5.79 s

$d_x = V_x t$

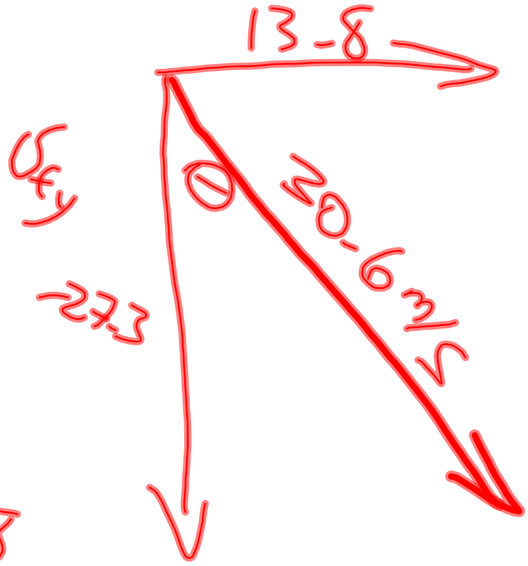
$= (13.8)(5.79) = \underline{79.9 \text{ m}}$

$V_f =$

$V_f = V_i + at$

$= 29.5 + (-9.81)(5.79)$

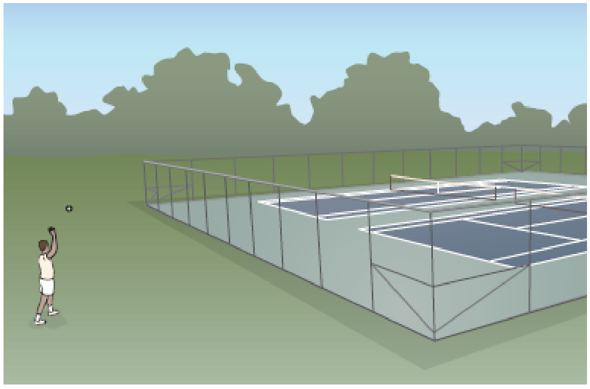
$= -27.3$

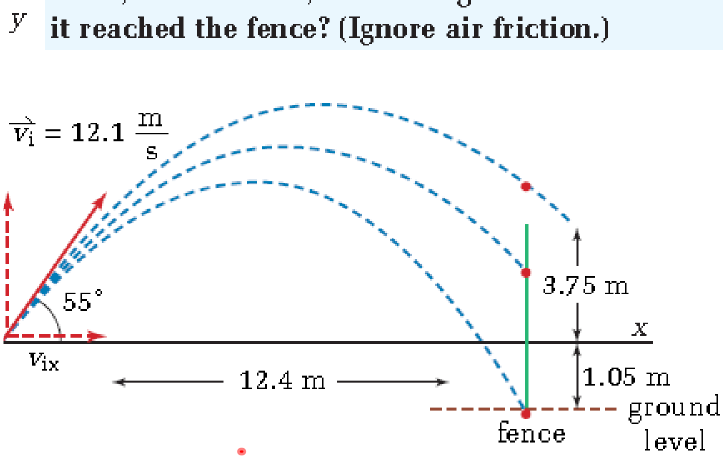


$\tan\theta = \frac{13.8}{27.3}$

$\theta = \underline{\underline{26.8°}}$

You are playing tennis with a friend on tennis courts that are surrounded by a 4.8 m fence. Your opponent hits the ball over the fence and you offer to retrieve it. You find the ball at a distance of 12.4 m on the other side of the fence. You throw the ball at an angle of 55.0° with the horizontal, giving it an initial velocity of 12.1 m/s. The ball is 1.05 m above the ground when you release it. Did the ball go over the fence, hit the fence, or hit the ground before it reached the fence? (Ignore air friction.)





Variables and Constants

Known		Implied	Unknown	
$\lvert\vec{v}_i\rvert = 12.1\ \frac{m}{s}$	$\Delta x = 12.4$ m	$a_y = -9.81\ \frac{m}{s^2}$	Δt	$\vec{v}_{iy}$
$\theta = 55°$	$h = 4.8$ m		$\vec{v}_{ix}$	Δy