10.4 Collisions and Explosions

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DEFINITION OF MOMENTUM

Momentum is the product of an object's mass and its velocity.

$$\vec{p} = m\vec{v}$$

Quantity	Symbol	SI unit
momentum	$\vec{p}$	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$ (kilogram metres per second)
mass	m	kg (kilograms)
velocity	$\vec{v}$	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$(\text{mass})(\text{velocity}) = \text{kg}\cdot\frac{\text{m}}{\text{s}} = \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Note: Momentum does not have a unique unit of its own.

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LAW OF CONSERVATION OF MOMENTUM

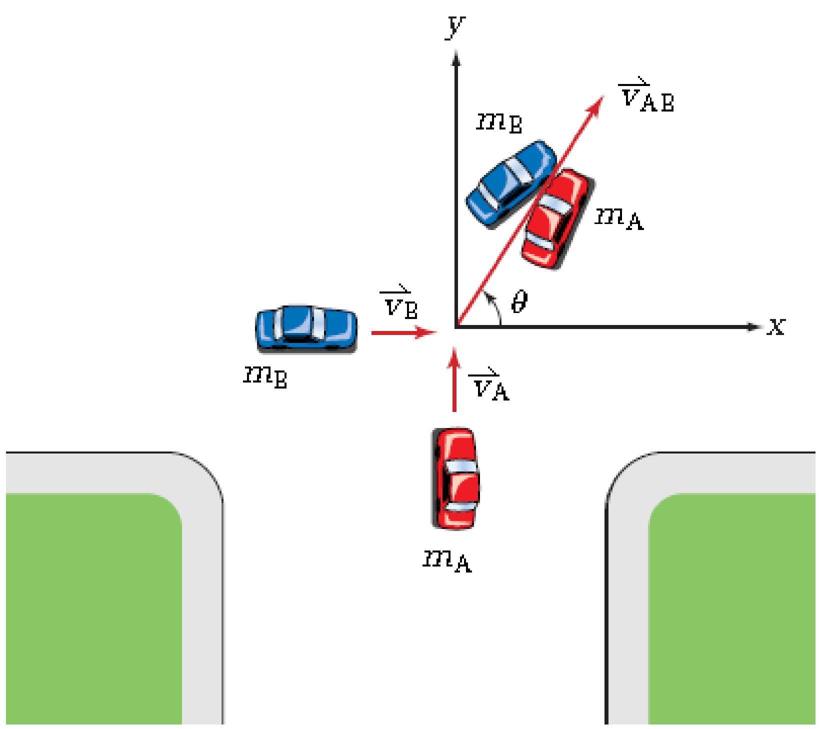
The sum of the momenta of two objects before collision is equal to the sum of their momenta after they collide.

$$\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$$

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

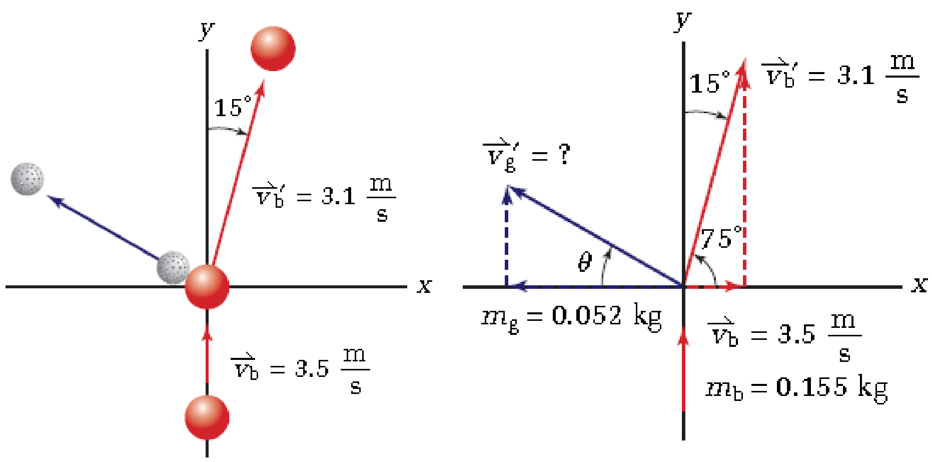
Quantity	Symbol	SI unit
mass of object A	m_A	kg (kilograms)
mass of object B	m_B	kg (kilograms)
velocity of object A before the collision	$\vec{v}_A$	$\frac{m}{s}$ (metres per second)
velocity of object B before the collision	$\vec{v}_B$	$\frac{m}{s}$ (metres per second)
velocity of object A after the collision	$\vec{v}'_A$	$\frac{m}{s}$ (metres per second)
velocity of object B after the collision	$\vec{v}'_B$	$\frac{m}{s}$ (metres per second)

Collisions in Two Dimensions



Applying Conservation of Momentum in Two Dimensions

A billiard ball of mass 0.155 kg is rolling directly away from you at 3.5 m/s. It collides with a stationary golf ball of mass 0.052 kg. The billiard ball rolls off at an angle of 15° clockwise from its original direction with a velocity of 3.1 m/s. What is the velocity of the golf ball?



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Frame the Problem

- Sketch the vectors representing the momentum of the billiard ball and the golf ball immediately before and just after the collision. It is always helpful to superimpose an *x*–*y*-coordinate system on the vectors so that the origin is at the point of the contact of the two balls. For calculations, use the angles that the vectors make with the x-axis.
- Momentum is *conserved* in the *x* and *y* directions *independently*.
- The *total momentum* of the system (billiard ball and golf ball) *before* the collision is carried by the *billiard ball* and is all in the positive *y* direction.
- *After* the collision, both balls have *momentum* in both the *y* direction and the *x* direction.

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Calculations

$$m_b v_{bx} + m_g v_{gx} = m_b v'_{bx} + m_g v'_{gx}$$

$$0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = m_b v'_{bx} + m_g v'_{gx}$$

$$m_g v'_{gx} = -m_b v'_{bx}$$

$$v'_{gx} = -\frac{m_b v'_{bx}}{m_g}$$

$$v'_{gx} = -\frac{(0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}} \cos 75°)}{0.052\ \text{kg}}$$

$$v'_{gx} = -2.3916\ \frac{\text{m}}{\text{s}}$$

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Calculations

$$m_b v_{by} + m_g v_{gy} = m_b v'_{by} + m_g v'_{gy}$$

$$m_b v_{by} + 0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = m_b v'_{by} + m_g v'_{gy}$$

$$m_g v'_{gy} = m_b v_{by} - m_b v'_{by}$$

$$v'_{gy} = \frac{m_b v_{by} - m_b v'_{by}}{m_g}$$

$$v'_{gy} = \frac{(0.155\ \text{kg})(3.5\ \frac{\text{m}}{\text{s}}) - (0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}} \sin 75°)}{0.052\ \text{kg}}$$

$$v'_{gy} = 1.507\ \frac{\text{m}}{\text{s}}$$

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$$|\vec{v}'_g|^2 = v'^2_{gx} + v'^2_{gy}$$

$$|\vec{v}'_g|^2 = \left(-2.3916\ \frac{m}{s}\right)^2 + \left(1.507\ \frac{m}{s}\right)^2$$

$$|\vec{v}'_g|^2 = 5.7198\ \frac{m^2}{s^2} + 2.271\ \frac{m^2}{s^2}$$

$$|\vec{v}'_g|^2 = 7.9908\ \frac{m^2}{s^2}$$

$$|\vec{v}'_g| = 2.8268\ \frac{m}{s}$$

$$|\vec{v}'_g| \cong 2.8\ \frac{m}{s}$$

$$\tan\theta = \frac{v'_{gy}}{v'_{gx}}$$

$$\tan\theta = \frac{1.507\ \frac{m}{s}}{2.3916\ \frac{m}{s}}$$

$$\theta = \tan^{-1} 0.6301$$

$$\theta = 32.22^\circ$$

$$\theta \cong 32^\circ$$