Universal gravitation

Short Answer

For the following questions, write the most appropriate answer in the space provided. Use the values given below if required.
$G = 6.673 \times 10^{-11}$ N•m^2/kg^2 and $M_{\text{Earth}} = 5.978 \times 10^{24}$ kg.

1. Find the speed of a satellite orbiting at a height of 100.0 km above the surface of Earth. $r_{\text{Earth}} = 6.378 \times 10^6$ m
2. The period of the Moon is 27.3 days. Find the radius of the of the Moon's orbit.
3. A synchronous communication satellite is a satellite that remains above the same location on Earth. Find the orbital radius of the satellite.
4. A space station in a circular orbit around the Moon has a speed of 1.57×10^3 m/s. Find the radius of its orbit. $M_{\text{Moon}} = 7.36 \times 10^{22}$ kg

Problem

Problem Solving
For the following questions, write the answer in the space provided. If the question requires mathematical calculations, show all of your work. Use the values given below if required.
$G = 6.673 \times 10^{-11}$ N•m^2/kg^2, $M_{\text{Earth}} = 5.978 \times 10^{24}$ kg, and $r_{\text{Earth}} = 6.378 \times 10^6$ m

5. A satellite is in a circular orbit at an altitude of 230 km above the surface of Earth.
   a) Find the period of the satellite.
   b) Is it a synchronous satellite?
6. A satellite of mass 550 kg is in a circular orbit at height 2.00×10^5 m above the surface of Earth. How much energy is required to double its height?

Universal gravitation
Answer Section

SHORT ANSWER

1. ANS:

$$\frac{GM_{\text{Earth}}M_{\text{Satellite}}}{r^2_{\text{orbit}}} = \frac{M_{\text{Satellite}}v^2}{r_{\text{orbit}}}$$

$$v = \sqrt{\frac{GM_{\text{Earth}}}{r_{\text{orbit}}}}$$

Radius of the orbit, $r_{\text{orbit}} = 6.378 \times 10^6 + 1.0 \times 10^5$
$= 6.478 \times 10^6$ m

Speed, $v = \sqrt{\frac{GM_{\text{Earth}}}{r_{\text{orbit}}}}$

$= \sqrt{\frac{(6.673 \times 10^{-11})(5.978 \times 10^{24})}{6.478 \times 10^{64}}}$

= 7847 m/s

DIF: easy REF: K/U

2. ANS:

$$\frac{GM_{\text{Earth}}M_{\text{Moon}}}{r^2_{\text{orbit}}} = \frac{4\pi^2 r_{\text{orbit}}M_{\text{Moon}}}{T^2}$$

$$\frac{GM_{\text{Earth}}}{r^2_{\text{orbit}}} = \frac{4\pi^2 r_{\text{orbit}}}{T^2}$$

$$\frac{(6.673 \times 10^{-11})(5.978 \times 10^{24})}{r^2_{\text{orbit}}} = \frac{4\pi^2 r_{\text{orbit}}}{(27.3 \times 24 \times 3600)^2}$$

The radius of the Moon’s orbit, $r_{\text{orbit}} = 3.84 \times 10^8$ m.

DIF: easy REF: K/U

3. ANS:

The period of a communication satellite is 24 h.

$$\frac{GM_{\text{Earth}}M_{\text{Satellite}}}{r^2_{\text{orbit}}} = \frac{4\pi^2 r_{\text{orbit}} M_{\text{Satellite}}}{T^2}$$

$$\frac{GM_{\text{Earth}}}{r^2_{\text{orbit}}} = \frac{4\pi^2 r_{\text{orbit}}}{T^2}$$

$$\frac{(6.673 \times 10^{-11})(5.978 \times 10^{24})}{r^2_{\text{orbit}}} = \frac{4\pi^2 r_{\text{orbit}}}{(24 \times 3600)^2}$$

Orbital radius: $r_{\text{orbit}} = 4.2 \times 10^7$ m

DIF: easy REF: K/U

4. ANS:

$$\frac{GM_{\text{Earth}}M_{\text{Satellite}}}{r^2_{\text{orbit}}} = \frac{M_{\text{Satellite}} v^2}{r_{\text{orbit}}}$$

$$\frac{GM_{\text{Earth}}}{r_{\text{orbit}}} = v^2$$

$$\frac{(6.673 \times 10^{-11})(7.36 \times 10^{22})}{r_{\text{orbit}}} = (1.57 \times 10^3)^2$$

$$r_{\text{orbit}} = 2.0 \times 10^6$$

Radius of the orbit: $r_{\text{orbit}} = 2.0 \times 10^6$ m.

DIF: easy REF: K/U

PROBLEM

5. ANS:

a) $$\frac{GM_{\text{Earth}}M_{\text{Satellite}}}{r^2_{\text{orbit}}} = \frac{4\pi^2 r_{\text{orbit}} M_{\text{Satellite}}}{T^2}$$

$$\frac{GM_{\text{Earth}}}{r^2_{\text{orbit}}} = \frac{4\pi^2 r_{\text{orbit}}}{T^2}$$

b) $$T = 2\pi\sqrt{\frac{r^3_{\text{orbit}}}{GM_{\text{Earth}}}}$$

Period: $$T = 2\pi\sqrt{\frac{(6.378 \times 10^6 + 2.3 \times 10^5)^3}{(6.673 \times 10^{-11})(5.978 \times 10^{24})}}$$

$= 5.344 \times 10^3$ s

DIF: easy REF: K/U

6. ANS:

Total energy at height = 200 km

$E_{total} = \dfrac{GM_{Earth}M_{Satellite}}{2r}$

$= -\dfrac{(6.673 \times 10^{-11})(5.978 \times 10^{24})(550)}{(2)(6.378 \times 10^{6} + 2 \times 10^{5})}$

$= -1.6934 \times 10^{10}$ J

Total energy at height = 400 km

$E_{total} = -\dfrac{(6.673 \times 10^{-11})(5.978 \times 10^{24})(550)}{(2)(6.378 \times 10^{6} + 4 \times 10^{5})}$

$= -1.6677 \times 10^{10}$ J

Energy required = $(-1.6677 \times 10^{10}) - (-1.6934 \times 10^{10})$

$= 2.57 \times 10^{8}$ J

DIF: average REF: K/U