Text p. 89, #5 Detailed Solution

5. Frame the Problem

- Let time zero be the moment that Michael begins to accelerate.
- At time zero, Michael is 75 m behind Robert and thus must run 75 m further than Robert in order to catch up with him.
- When Michael catches up to Robert, they will have run for the same amount of time.
- Michael is travelling with uniform acceleration. Thus, the equation of motion that relates displacement, initial velocity, acceleration, and time interval describes Michael's motion.
- Robert travels with constant velocity or uniform motion. Robert's motion can therefore be described by using the equation that defines velocity.

Variables and Constants

Known	Unknown
$a_M = 0.15$ m/s^2	$\Delta t = ?$
$v_{M(i)} = 3.8$ m/s	Δd_M
$v_R = 4.2$ m/s	$\Delta d_R + 75$ m

Calculations

$$\Delta d_M = \Delta d_R + 75 \text{ m}$$

$$v_M\Delta t + \frac{1}{2}a_M\Delta t^2 = 75 \text{ m} + v_R\Delta t$$

$$3.8\ \frac{\text{m}}{\text{s}}\Delta t + \frac{1}{2}\left(0.15\ \frac{\text{m}}{\text{s}^2}\right)\Delta t^2 = 75 \text{ m} + 4.2\ \frac{\text{m}}{\text{s}}\Delta t$$

$$3.8\ \frac{\text{m}}{\text{s}}\Delta t + \left(0.075\ \frac{\text{m}}{\text{s}^2}\right)\Delta t^2 = 75 \text{ m} + 4.2\ \frac{\text{m}}{\text{s}}\Delta t$$

$$3.8\ \frac{\text{m}}{\text{s}}\Delta t - 4.2\ \frac{\text{m}}{\text{s}}\Delta t + \left(0.075\ \frac{\text{m}}{\text{s}^2}\right)\Delta t^2 - 75 = 0$$

$$-0.4\ \frac{\text{m}}{\text{s}}\Delta t + \left(0.075\ \frac{\text{m}}{\text{s}^2}\right)\Delta t^2 - 75 = 0$$

$$\left(0.075\ \frac{\text{m}}{\text{s}^2}\right)\Delta t^2 - 0.4\ \frac{\text{m}}{\text{s}}\Delta t - 75 = 0$$

Strategy

Write a mathematical equation that states that the distance Michael runs is equal to the distance Robert runs during the time interval, plus the 75 m Michael has to make up.

Substitute the equation that defines the velocity for Robert for Δd_R. Substitute the equation of motion that relates displacement, initial velocity, acceleration, and the time interval for Michael for Δd_M.

The time interval from time zero is the same for the two runners when Michael catches up with Robert. Solve for Δt, the unknown, after substituting the known values into the equations.

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta t = \frac{-(-0.4) \pm \sqrt{(-0.4)^2 - 4(0.075)(-75)}}{2(0.075)}$$

$$\Delta t = \frac{0.4 \pm \sqrt{0.16 + 22.5}}{0.15}$$

$$\Delta t = \frac{0.4 \pm \sqrt{22.66}}{0.15}$$

$$= \frac{0.4 \pm \sqrt{4.76}}{0.15}$$

$$\Delta t = \frac{0.4 + 4.76}{0.15} \qquad \Delta t = \frac{0.4 - 4.76}{0.15}$$

$$\Delta t = \frac{5.16}{0.15} \qquad \Delta t = \frac{-4.36}{0.15}$$

$$\therefore \Delta t = 34.4 \text{ s} \qquad \therefore \Delta t = -29.07 \text{ s}$$

Use the quadratic formula to solve for Δt, since it cannot be easily factored.

Exclude –29.07, since a negative time has no meaning in this situation.

It will take Michael 34 s to catch Robert.