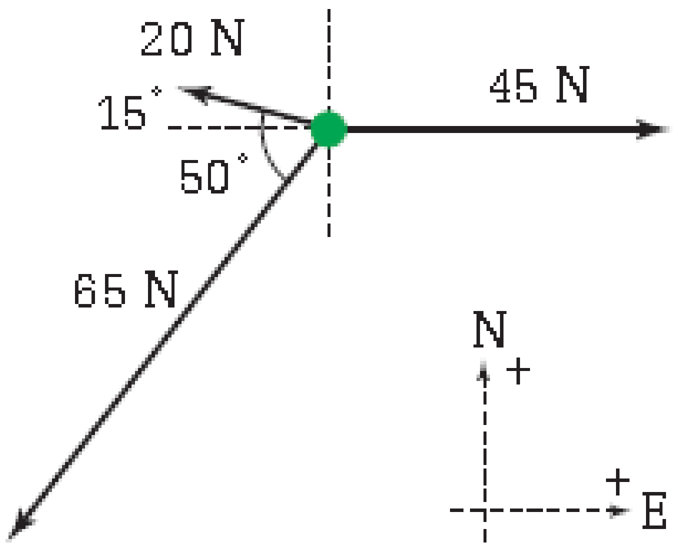
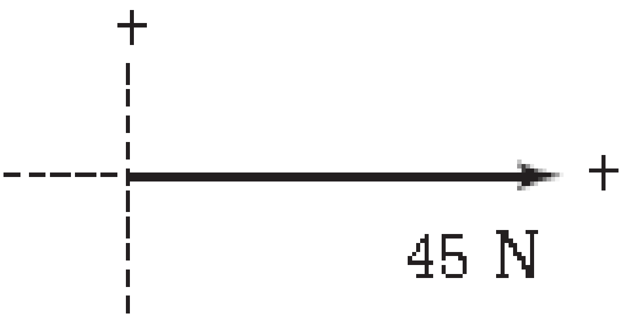
Three children are each pulling on their older sibling, who has a mass of 65 kg. The forces exerted by each child are listed below. Use vector components to find the acceleration of the older sibling.



Solution

- **Determine the algebraic sum of the X and Y components of each vector to determine the resultant force vector**

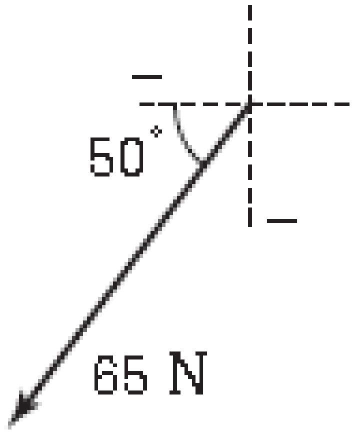
F_1



$$F_{1x} = 45\ \mathrm{N}$$

$$F_{1y} = 0.0\ \mathrm{N}$$

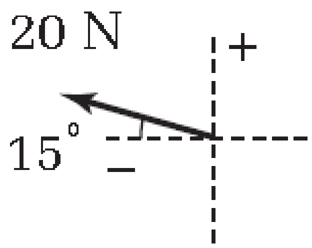
F_2



$$F_{2x} = -|\overrightarrow{F_2}| \cos 50^\circ$$
$$F_{2x} = -(65\text{ N})(0.6428)$$
$$F_{2x} = -41.78\text{ N}$$

$$F_{2y} = -|\overrightarrow{F_2}| \sin 50^\circ$$
$$F_{2y} = -(65\text{ N})(0.7660)$$
$$F_{2y} = -49.79\text{ N}$$

F_3



$$F_{3x} = -|\overrightarrow{F_3}| \cos 15^\circ$$
$$F_{3x} = -(20\text{ N})(0.9659)$$
$$F_{3x} = -19.32\text{ N}$$

$$F_{3y} = |\overrightarrow{F_3}| \sin 15^\circ$$
$$F_{3y} = (20\text{ N})(0.2588)$$
$$F_{3y} = 5.176\text{ N}$$

Collect Components in a table

Vector	*x*-component	*y*-component
$\vec{F}_1$	45 N	0.0 N
$\vec{F}_2$	−41.78 N	−49.79 N
$\vec{F}_3$	−19.32 N	5.176 N
$\vec{F}_{net}$	−16.1 N	−44.614 N

Magnitude

$$|\vec{F}_{net}|^2 = (F_{x\ net})^2 + (F_{y\ net})^2$$

$$|\vec{F}_{net}|^2 = (-16.1\ N)^2 + (-44.614\ N)^2$$

$$|\vec{F}_{net}|^2 = 259.21\ N^2 + 1990.41\ N^2$$

$$|\vec{F}_{net}|^2 = 2249.62\ N^2$$

$$|\vec{F}_{net}| = 47.430\ N$$

Direction

$$\tan\theta = \frac{-44.614\ N}{-16.1\ N}$$

$$\tan\theta = 2.7711$$

$$\theta = \tan^{-1} 2.7711$$

$$\theta = 70.16^\circ$$

Recall

Use vector components to find the acceleration of the older sibling.

$$\vec{a} = \frac{\vec{F}}{m}$$

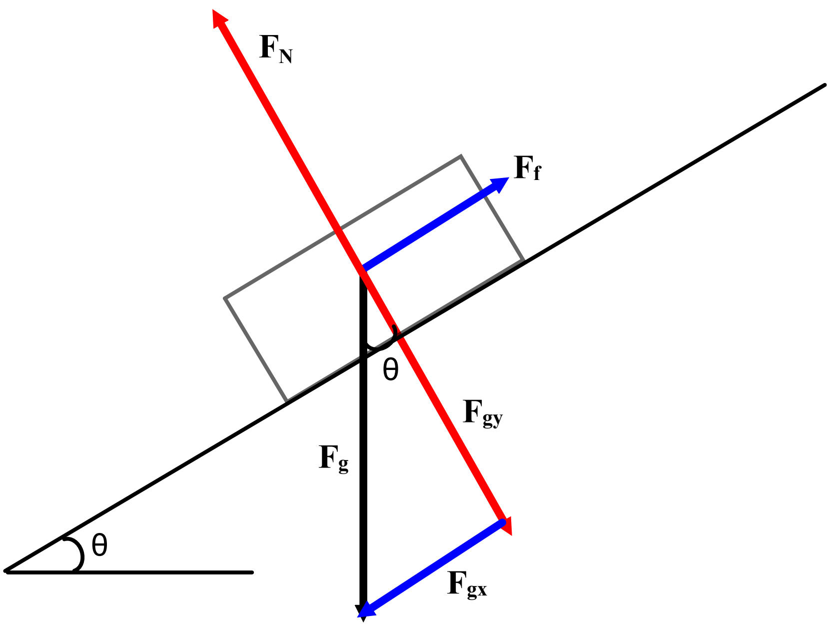
$$\vec{a} = \frac{47.43\ \text{N}[\text{S}20^\circ\text{W}]}{65\ \text{kg}}$$

$$\vec{a} = 0.72969\ \frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{\text{kg}}[\text{S}20^\circ\text{W}]$$

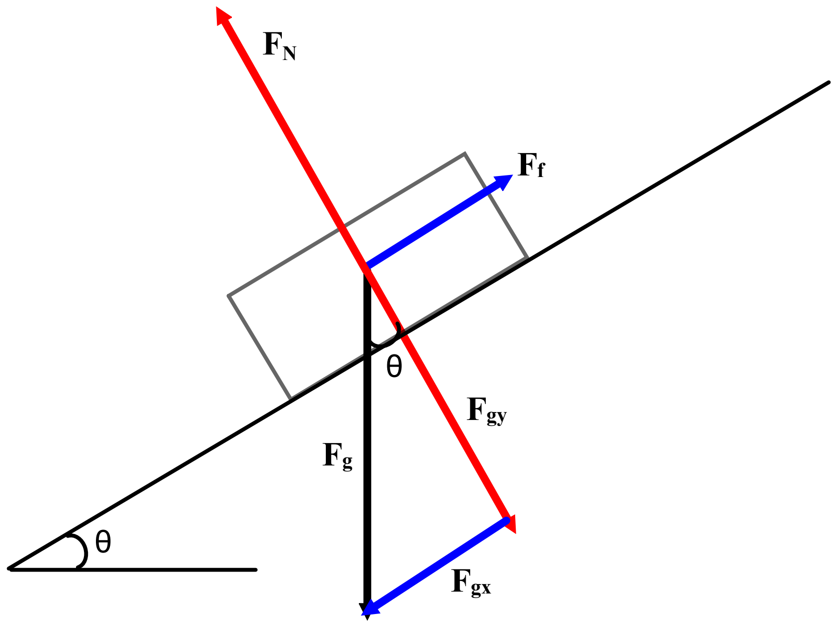
$$\vec{a} = 0.79269\ \frac{\text{m}}{\text{s}^2}[\text{S}20^\circ\text{W}]$$

The acceleration of the older sibling is $0.73\ \frac{\text{m}}{\text{s}^2}[\text{S}20^\circ\text{W}]$.

Inclined Planes



Inclined Planes



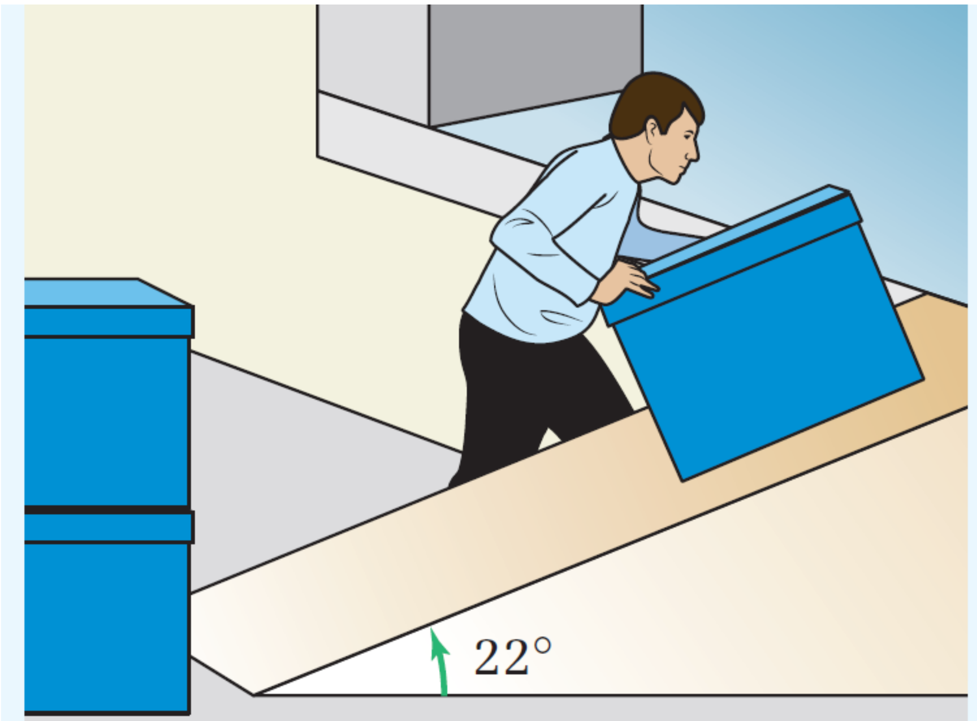
A worker places a large plastic waste container with a mass of 84 kg on the ramp of a loading dock as shown in the figure.

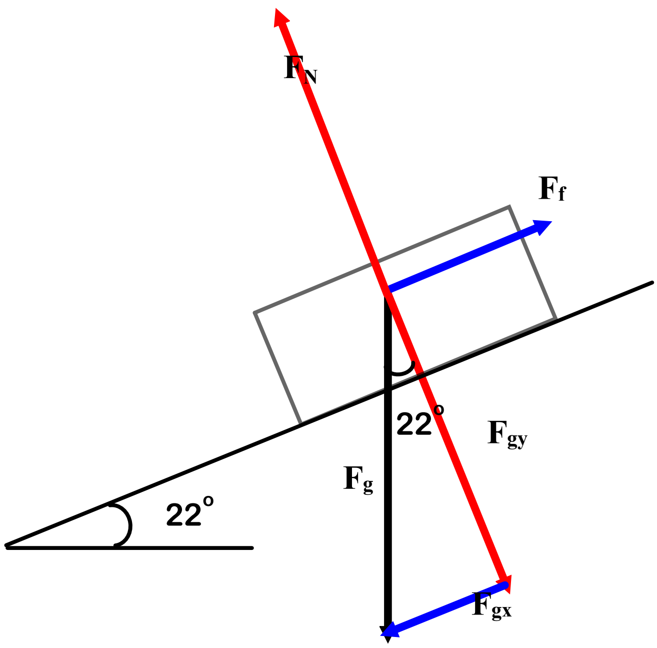
The ramp makes an angle of 22° with the horizontal. The workers turns to pick up another container before pushing the first one up the ramp.

(a) If the coefficient of static friction is 0.47, will the crate slide down the ramp?

(b) If the crate does slide down, what will be its acceleration?

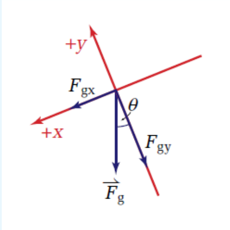
(c) If the crate does not slide down, and the worker starts to push it up the ramp with an applied force parallel to the plane, with what force will he be pushing when the crate begins to move?


22°


F_N
F_f
22°
F_{gy}
F_g
22°
F_{gx}

Does the box slide down the ramp?

Strategy	Calculations
Find the *x* component of the force of gravity.	$\sin\theta = \frac{F_{gx}}{\lvert\vec{F}_g\rvert}$ $F_{gx} = \lvert\vec{F}_g\rvert \sin\theta$ $F_{gx} = mg\sin\theta$ $F_{gx} = (84\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 22^\circ$ $F_{gx} = (824.04\ \text{N})(0.374607)$ $F_{gx} = 308.69\ \text{N}$
Find the maximum possible force of static friction.	$F_{f(max)} = \mu_s F_N$ $F_{f(max)} = \mu_s mg\cos\theta$ $F_{f(max)} = (0.47)(84\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\cos 22^\circ$ $F_{f(max)} = (387.29\ \text{N})(0.92718)$ $F_{f(max)} = 359.097\ \text{N}$



Compare F_{gx} with $F_{f(max)}$

What force must be applied to get the box to move UP the ramp?

Find the applied force by applying Newton's second law to the components of force in the *x* direction. The force of friction will be positive (down the ramp) because the applied force is in the negative direction (up the ramp).

$$F_x = ma_x$$
$$F_a + F_{gx} + F_f = ma_x$$
$$F_a + mg\sin\theta + \mu_s F_N = ma_x$$

The acceleration in the x direction will be zero until the applied force has overcome the force of static friction.

$$F_a + mg\sin\theta + \mu_s F_N = 0$$
$$F_a = -mg\sin\theta - \mu_s F_N$$
$$F_a = -(84\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 22^\circ - (0.47)(764.04\text{ N})$$
$$F_a = -(824.04\text{ N})(0.37461) - (359.09\text{ N})$$
$$F_a = -308.69\text{ N} - 359.09\text{ N}$$
$$F_a = -667.787\text{ N}$$
$$F_a \cong -6.7\times 10^2\text{ N}$$

Tension in Ropes and Cables

SECTION OUTCOMES

- Use vector analysis in two dimensions for systems involving two or more masses.
- Analyze systems of two or more masses including inclined planes.
- Use Newton's laws of motion and the concepts of frictional forces and normal forces.

KEY TERMS

- tension
- counterweight
- system
- internal forces
- external forces

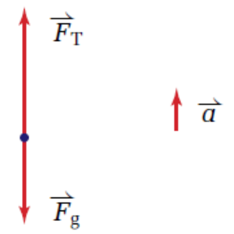
MODEL PROBLEM

Tension in a Cable

An elevator filled with people has a total mass of 2245 kg. As the elevator begins to rise, the acceleration is 0.55 m/s^2. What is the tension in the cable that is lifting the elevator?

Strategy

Apply Newton's second law and insert all of the forces acting on the elevator. Then solve for the tension.



What force (F_T) must be supplied to overcome the weight of the car AND cause the acceleration indicated?

$$F_T = F_g + ma$$