Notes#12 Solving Absolute Value Equations

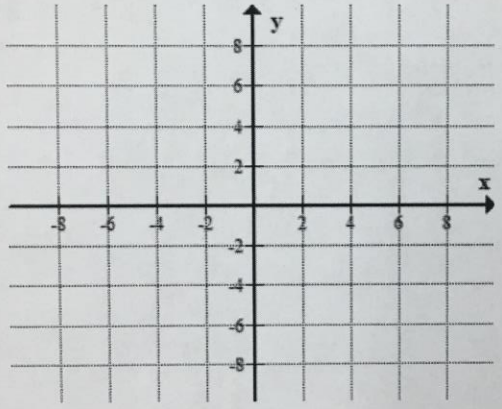
In this lesson we will be solving **absolute value equations**.

They can be solved graphically or algebraically. The first example shows both methods. The rest of the examples will be solved algebraically only.

Solving Absolute Value Equations:

Solve the equation $|3 - 2x| = 6$ *for* x

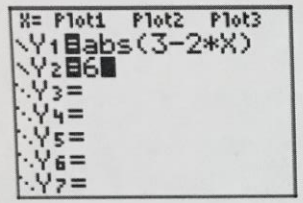
- Two strategies: Graphical Solution or Algebraic Solution

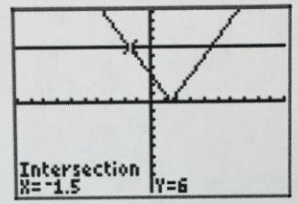


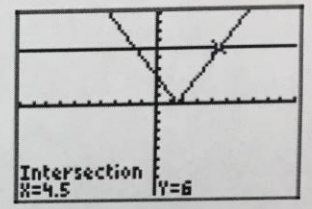
Using graphic calculator

Next slide

Graphical Strategy:(using graphic calculator)







check

Answers
-1.5 and 4.5

Verify
$|3-2(-1.5)| = 6$
$|3+3| = 6$
$|6| = 6$
$6 = 6$ ✓

Verify
$|3-2(4.5)| = 6$
$|3-9| = 6$
$|-6| = 6$
$6 = 6$ ✓

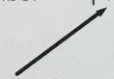
Algebraic Strategy:

Case 1: expression inside absolute value symbol is positive or zero

Case 2 : inside is negative

You must always check the **positive** case and **negative** case

Remember $|?| = 5$



Two answers could be +5 or -5

$$|3-2x|=6$$

positive case

$+(3-2x)=6$

$3-2x=6$

$-2x=3$

$x=\frac{3}{-2}=-1.5$

Verify

$|3-2(-1.5)|=6$

$|3+3|=6$

$|6|=6$

$6=6$ ✓

negative case

$-(3-2x)=6$

$-3+2x=6$

$2x=9$

$x=4.5$

Verify

$|3-2(4.5)|=6$

$|3-9|=6$

$|-6|=6$

$6=6$ ✓

Answers

$x=-1.5$ and 4.5

Example

$$|6 - x| = 2$$

positive case

$+(6-x) = 2$

$6 - x = 2$

$-x = -4$

$x = 4$

Verify

$|6 - 4| = 2$

$|2| = 2$

$2 = 2$ ✓

negative case

$-(6-x) = 2$

$-6 + x = 2$

$x = 8$

Verify

$|6 - 8| = 2$

$|-2| = 2$

$2 = 2$ ✓

Answers

$x = 4$ and 8

Absolute Value Equation With an Extraneous Solution

(invalid solution)

Solve $|2x - 5| = 5 - 3x$.

** always verify

positive case

$+(2x-5) = 5-3x$

$2x - 5 = 5 - 3x$

$5x = 10$

$x = 2$

Verify

$|2(2)-5| = 5-3(2)$

$|4-5| = 5-6$

$|-1| = -1$

$1 = -1$

no

negative case

$-(2x-5) = 5-3x$

$-2x+5 = 5-3x$

$x = 0$

Verify

$|2(0)-5| = 5-3(0)$

$|-5| = 5$

$5 = 5$ ✓

Answer

$x = 0$

Absolute Value Equation With No Solution

Solve $|3x - 4| + 12 = 9$.

$|3x-4| = 9-12$

$|3x-4| = -3$

absolute value can not equal a negative number

What about a quadratic absolute value equation??

Your Turn

Solve $|x^2 - 3x| = 2$.

positive case

$+(x^2-3x) = 2$

$x^2 - 3x - 2 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2-4(1)(-2)}}{2}$

$= \frac{3 \pm \sqrt{9+8}}{2}$

$= \frac{3 \pm \sqrt{17}}{2}$

$\approx \frac{3 \pm 4.12}{2}$

≈ 3.56 and -0.56

negative case

$-(x^2-3x) = 2$

$-x^2+3x = 2$

$0 = x^2 - 3x + 2$

$0 = (x-1)(x-2)$

$x = 1, 2$

Verify

$|(3.56)^2 - 3(3.56)| = 2$

$|2| = 2$

$2 = 2$ ✓

Verify

$|(-0.56)^2 - 3(-0.56)| = 2$

$|2| = 2$

$2 = 2$ ✓

Verify

$|(1)^2 - 3(1)| = 2$

$|-2| = 2$

$2 = 2$ ✓

Verify

$|(2)^2 - 3(2)| = 2$

$|4-6| = 2$

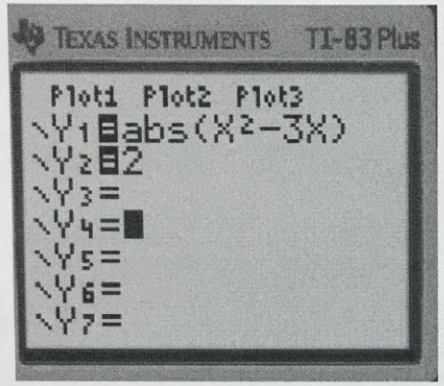
$|-2| = 2$

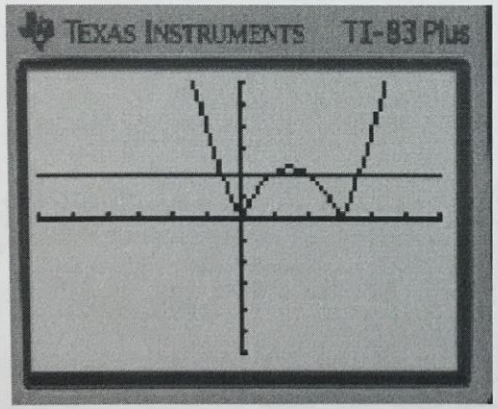
$2 = 2$ ✓

Answer

$x = 1, 2, 3.56, -0.56$

Check on calculator





4 answers

$x = 1, 2, 3.56, -0.56$

Solve an Absolute Value Equation Involving Linear and Quadratic Expressions

Solve $|x - 10| = x^2 - 10x$.

positive case

$+(x-10) = x^2-10x$

$x-10 = x^2-10x$

$0 = x^2-11x+10$

$0 = (x-10)(x-1)$

$x = 1, 10$

negative case

$-(x-10) = x^2-10x$

$-x+10 = x^2-10x$

$0 = x^2-9x-10$

$0 = (x-10)(x+1)$

$x = -1, 10$

Verify $(x=1)$

$|1-10| = (1)^2-10(1)$

$|-9| = 1-10$

$|-9| = -9$

$9 = -9$ no

Verify $(x=-1)$

$|-1-10| = (-1)^2-10(-1)$

$|-11| = 1+10$

$11 = 11$ ✓

Verify $(x=10)$

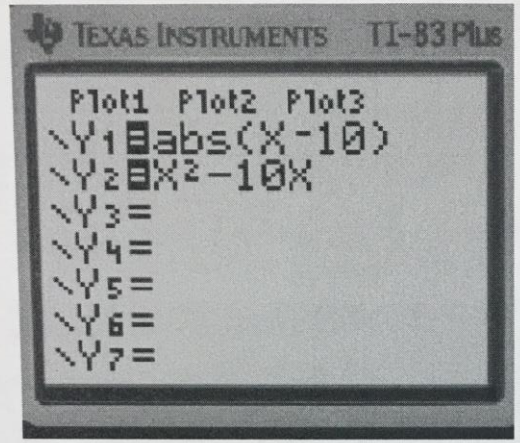
$|10-10| = (10)^2-10(10)$

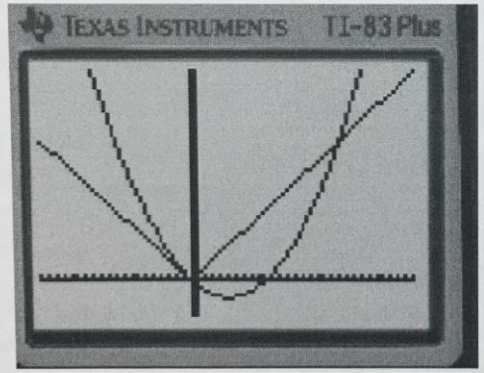
$|0| = 100-100$

$0 = 0$ ✓

Answers

$x = 10, -1$





Answers

$x = 10, -1$