Notes#13 L’Hospital’s Rule

In this lesson we will look at **L'Hospital's Rule**.

This is a rule you can use to find limits.

I could not show you this rule when we took the limit section in Pre-Cal12B because this rule involves derivatives which we had not yet covered.

L'Hopital's Rule

If $\lim_{x \to a} \frac{f(x)}{g(x)}$ results in the indeterminate

If you fill in for x and get $\longrightarrow$ $\frac{0}{0}$ or $\frac{\infty}{\infty}$

then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Ex. Find the limit

$$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 5x + 6}$$

How we did it in Pre-Cal12B

1. Fill in get $\frac{0}{0}$ (indeterminate form)
2. Factor
3. Solve

$$\lim_{x \to 3} \frac{(x-3)(x^2+3x+9)}{(x-3)(x-2)}$$

$$= \frac{(3)^2 + 3(3) + 9}{3-2}$$

$$= \frac{9+9+9}{1} = \boxed{27}$$

Option because it is **indeterminate** you could use **L'Hospital's Rule**

$$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 5x + 6}$$

1. Take **derivative** of top and bottom
2. Solve

$$\lim_{x \to 3} \frac{3x^2}{2x - 5}$$

$$= \frac{3(3)^2}{2(3) - 5}$$

$$= \frac{27}{1} = \boxed{27}$$

Ex. Find the limit

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$$= \frac{3(3)^2}{2(3) - 5}$$

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$\left(\frac{0}{0}\right)$

Ex. $\lim_{x \to 0} \frac{\sin x}{x}$

$\lim_{x \to 0} \frac{\cos x}{1}$

$= \cos 0$

$= 1$

$$\lim_{x \to \infty} \frac{\ln x}{x} \qquad \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x \to \infty} \frac{\frac{1}{x}}{1}$$

$$= \frac{0}{1} = \boxed{0}$$

$$\lim_{x \to \infty} \frac{e^x}{x^2} \qquad \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x \to \infty} \frac{e^x}{2x} \qquad \text{still } \left(\frac{\infty}{\infty}\right)$$

Take derivative again

$$\lim_{x \to \infty} \frac{e^x}{2}$$

$$= \frac{\infty}{2}$$

$$= \infty$$

$$\lim_{x\to\infty} \frac{x}{\ln(1+2e^x)} \qquad \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x\to\infty} \frac{1}{\frac{1}{1+2e^x}\cdot 2e^x}$$

$$\lim_{x\to\infty} \frac{1}{\frac{2e^x}{1+2e^x}}$$

$$\lim_{x\to\infty} \frac{1+2e^x}{2e^x}$$

still $\left(\frac{\infty}{\infty}\right)$

take derivative again

$$\lim_{x\to\infty} \frac{2e^x}{2e^x}$$

$$= \boxed{1}$$