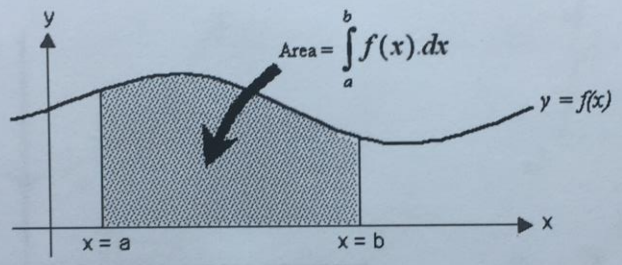
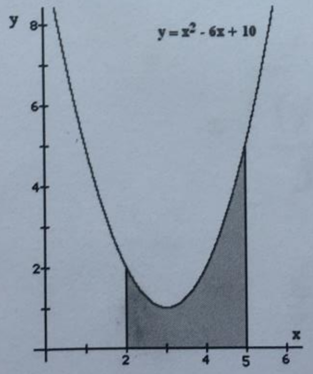
Notes#11 Area Under the Curve

We will now use the integration techniques you have learned to find area under curves and area between curves. Area under the curve can represent many different things, some you would be familiar with from physics would be displacement, velocity and work.

Area Under a Curve



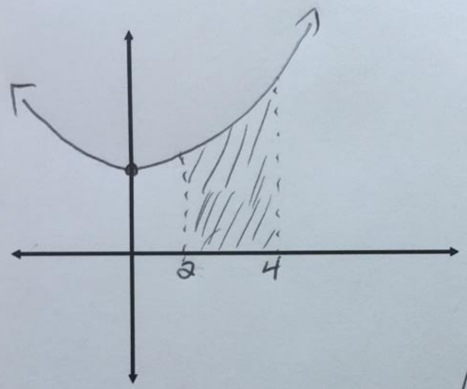


Example 1 (Easiest type because the boundaries are given)

lower limit

Find the area under the curve $y = x^2+2$ from 2 to 4. (note and above the x-axis).

upper limit



$$\int_2^4 x^2+2 \, dx$$

$$= \frac{x^3}{3} + 2x \Big|_2^4$$

$$= \left[\frac{(4)^3}{3} + 2(4)\right] - \left[\frac{(2)^3}{3} + 2(2)\right]$$

$$= \frac{64}{3} + 8 - \frac{8}{3} - 4$$

$$= \frac{56}{3} + 4$$

$$= 18\frac{2}{3} + 4$$

$$= 22\frac{2}{3} \ u^2$$

Example 2 (Be careful, if the area between the curve and the y-axis is required, you must rearrange the equation for x = and integrate using y.)

Y-Axis Boundaries

Determine the area of the region bound by $y = \sqrt{x}$ $[x = y^2]$ and the y-axis from y = 0 to y = 4.

y -axis boundaries

$$\int_c^d g(y)\,dy$$

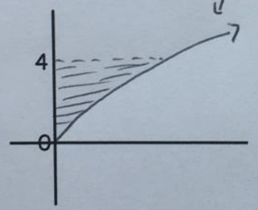
g(y) is a function in the form x =

x -axis boundaries

$$\int_a^b f(x)\,dx$$

f(x) is a function in the form y =

$y = \sqrt{x}$ or $x = y^2$



$$A = \int_c^d g(y)\,dy$$

$$\int_0^4 y^2\,dy$$

$$= \left.\frac{y^3}{3}\right|_0^4$$

$$= \frac{(4)^3}{3} - \frac{(0)^3}{3}$$

$$= \frac{64}{3} - 0$$

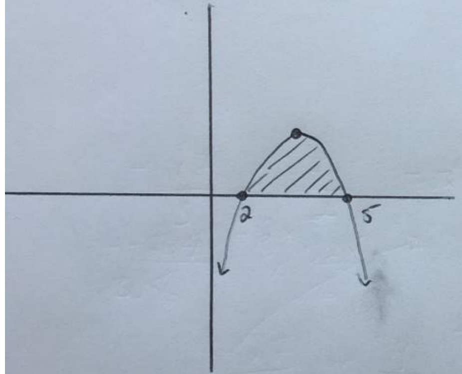
$$= \frac{64}{3}\,u^2$$

Example 3 (The upper and lower limits are not provided, you must find them first.)

Determine the area of the region bound by $y = -x^2+7x-10$ and the x-axis.
$(y=0)$

* find points on the axis by setting it equal to zero

$$0 = -x^2 + 7x - 10$$
$$0 = x^2 - 7x + 10$$
$$0 = (x-2)(x-5)$$
$$x = 2, 5$$



$$\int_2^5 -x^2 + 7x - 10 \, dx$$

$$= -\frac{x^3}{3} + \frac{7x^2}{2} - 10x \Big|_2^5$$

$$= \left[-\frac{(5)^3}{3} + \frac{7(5)^2}{2} - 10(5) \right] - \left[-\frac{(2)^3}{3} + \frac{7(2)^2}{2} - 10(2) \right]$$

$$= -\frac{125}{3} + \frac{175}{2} - 50 + \frac{8}{3} - 14 + 20$$

$$= -\frac{117}{3} + \frac{175}{2} - 44$$

$$= -\frac{234}{6} + \frac{525}{6} - \frac{264}{6} \rightarrow \boxed{\frac{27}{6} = \frac{9}{2} = 4\frac{1}{2}\, u^2}$$

Example 4 (Area between two curves, new formula)

Formula

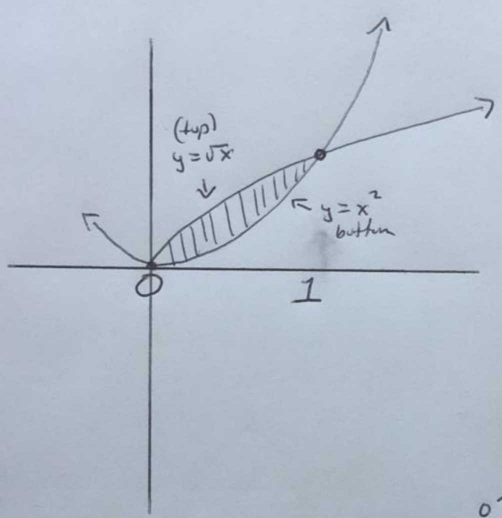
<u>Area between curves</u>: (x-axis as limits)

$$\int_a^b [\, f(x) - g(x) \,]\, dx$$

top curve (f(x)); bottom curve (g(x))

Example 4

Find the area between $y = x^2$ and $y = \sqrt{x}$.



$$x^2 = \sqrt{x}$$
$$(x^2)^2 = (\sqrt{x})^2$$
$$x^4 = x$$
$$x^4 - x = 0$$
$$x(x^3 - 1) = 0$$
$$\boxed{x = 0, 1}$$

$$\int_0^1 (\sqrt{x}) - (x^2)\, dx$$

$$= \int_0^1 x^{\frac{1}{2}} - x^2\, dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\Big|_0^1$$

$$= \left[\frac{2}{3}(1)^{\frac{3}{2}} - \frac{(1)^3}{3}\right] - \left[\frac{2}{3}(0)^{\frac{3}{2}} - \frac{(0)^3}{3}\right]$$

$$= \left[\frac{2}{3}(\sqrt{1})^3 - \frac{1}{3}\right] - 0$$

$$= \frac{2}{3} - \frac{1}{3}$$

$$= \frac{1}{3}\ u^2$$