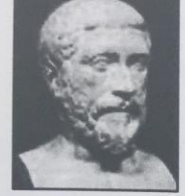
Notes#7 Reference Triangles

In this lesson we will review right angle trigonometry.

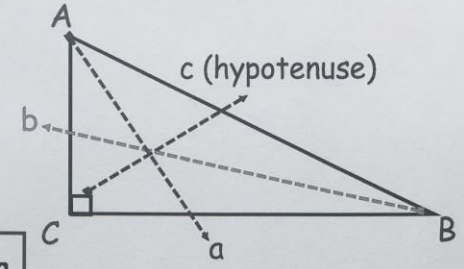
The new work will include using reference triangles to find primary trigonometric ratios,reference angles and rotation angles.

Also we will look at $\sin\theta$, $\cos\theta$ and $\tan\theta$ in terms of x,y,and r.



Pythagorean Theorem

- is a fundamental relationship amongst the sides on a **RIGHT triangle**.

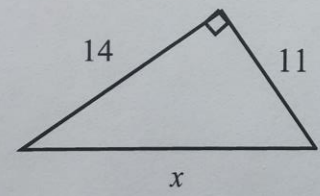


$$c^2 = a^2 + b^2$$

OPTIONS...

#1. Finding the unknown hypotenuse:

ex:



$$c^2 = a^2 + b^2$$

$x^2 = 14^2 + 11^2$

$x^2 = 196 + 121$

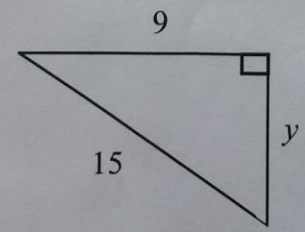
$\sqrt{x^2} = \sqrt{317}$

$x \approx 17.8$

#2. Finding an unknown side

$$a^2 = c^2 - b^2$$

ex:



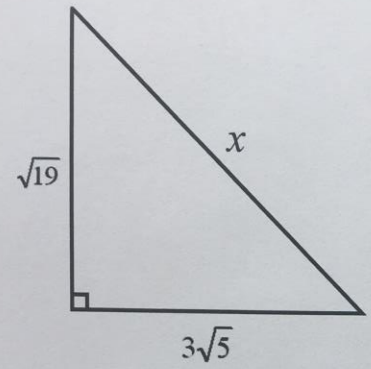
$y^2 = 15^2 - 9^2$

$y^2 = 225 - 81$

$\sqrt{y^2} = \sqrt{144}$

$y = 12$

Check Up...



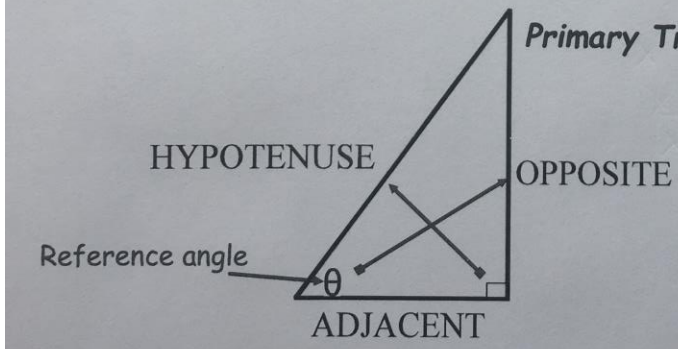
$$x^2 = (\sqrt{19})^2 + (3\sqrt{5})^2$$

$$x^2 = 19 + 45$$

$$\sqrt{x^2} = \sqrt{64}$$

$$x = 8$$

Trigonometric Ratios



Primary Trigonometric Ratios

$$\sin\theta = \frac{opp}{hyp}$$

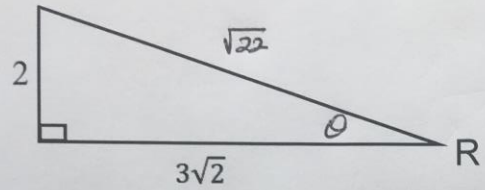
$$\cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj}$$

Memory Aid: "SOH CAH TOA"

Check up...

State the three primary trigonometric ratios of angle R. Express your ratios as fractions. (Do not have to be in simplest form)



$c^2 = 2^2 + (3\sqrt{2})^2$

$c^2 = 4 + 18$

$\sqrt{c^2} = \sqrt{22}$

$c = \sqrt{22}$

$\sin\theta = \frac{2}{\sqrt{22}}$ $\cos\theta = \frac{3\sqrt{2}}{\sqrt{22}}$ $\tan\theta = \frac{2}{3\sqrt{2}}$

Evaluate each of the following:

$\sin 78^\circ =$ _____

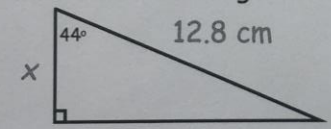
sin(78)
.9781476007

$\cos\theta = 0.6469$

$\theta =$ $\cos^{-1}$ .6469

$\theta \approx 50^\circ$

EXAMPLE - Finding an unknown side

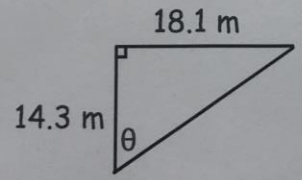


$\cos 44^\circ = \frac{x}{12.8}$

$x = (\cos 44^\circ)(12.8)$

$x \approx 9.2$ cm

EXAMPLE - Finding an unknown angle



$\tan\theta = \frac{18.1}{14.3}$

$\tan\theta \approx 1.2657$

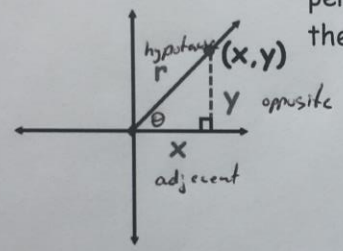
$\theta = \tan^{-1} 1.2657$

$\theta \approx 52^\circ$

Now that the review is over we are going to use reference triangles to find trigonometric values.

Angles on the Cartesian Plane

- **Reference Angle** - an acute angle formed between the terminal arm and the *x*-axis.
- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the *x*-axis.



Primary Trigonometric Ratios (in terms of x, y, r)

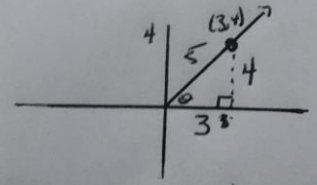
$$\sin\theta = \frac{y}{r} = \frac{O}{H}$$

$$\cos\theta = \frac{x}{r} = \frac{A}{H}$$

$$\tan\theta = \frac{y}{x} = \frac{O}{A}$$

Ex. The terminal arm of an angle passes through the point (3,4).

Plot the point, over 3, up 4.



The point (3,4) represents two sides of the reference triangle.

(3,4)
x y
↑ ↖
adjacent opposite

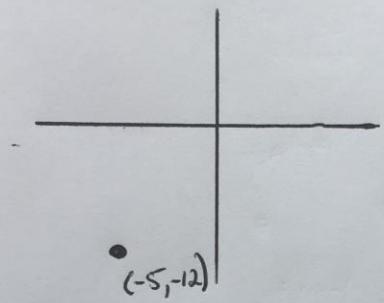
$r^2 = 3^2 + 4^2$
$r^2 = 9 + 16$
$\sqrt{r^2} = \sqrt{25}$
$r = 5$

Example 1

The terminal arm of angle θ passes through the point, P(-5,-12).

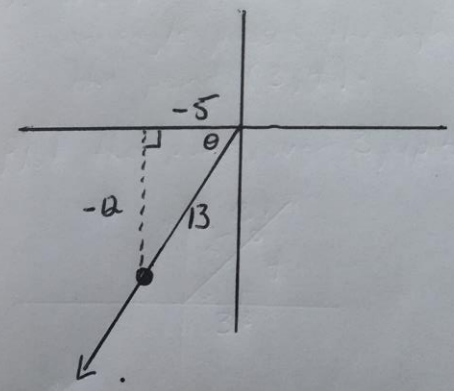
Find the three primary trigonometric ratios.

Step 1 Plot (-5, -12), so over -5 down -12 (3rd quadrant)



x ← adjacent
y ← opposite

Step 2 Draw terminal arm through (-5, -12)
Next draw reference triangle (always to x-axis)



Step 3 Find hypotenuse

$c^2 = (-12)^2 + (-5)^2$

$c^2 = 144 + 25$

$\sqrt{c^2} = \sqrt{169}$

$c = 13$

answers

$\sin\theta = \frac{-12}{13}$

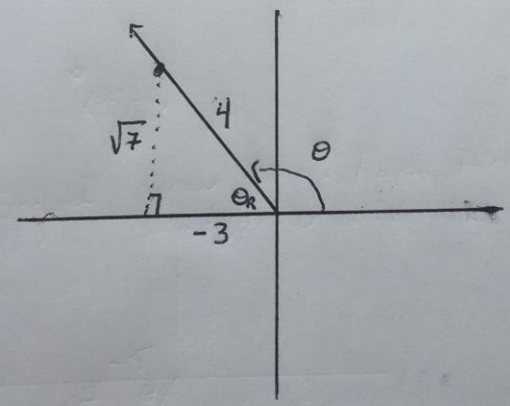
$\cos\theta = \frac{-5}{13}$

$\tan\theta = \frac{-12}{-5} = \frac{12}{5}$

Example 2

A positive angle, θ, is in the second quadrant. If $\cos\theta = \frac{-3}{4}$, find the value of the other two primary trigonometric ratios.

$\cos\theta = \frac{-3}{4} = \frac{X}{r} = \frac{A}{H}$ ← ratio gives two sides of your reference triangle



$y^2 = 4^2 - (-3)^2$
$y^2 = 16 - 9$
$\sqrt{y^2} = \sqrt{7}$
$y = \sqrt{7}$

Answer

$\sin\theta = \frac{\sqrt{7}}{4}$ $\tan\theta = -\frac{\sqrt{7}}{3}$