Notes#8 Finding Multiple Angles(CAST Rule)

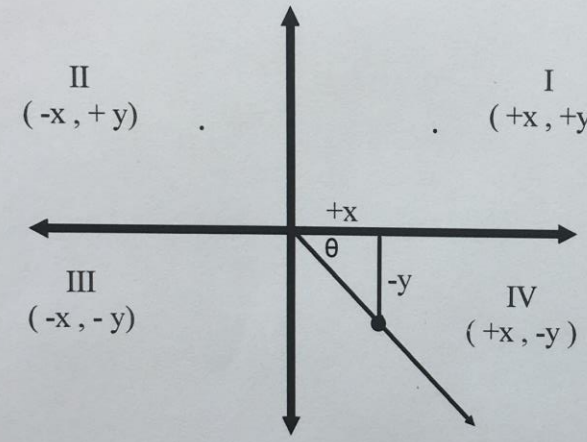
Find Multiple Angles

In this lesson we are going to find multiple angles from a single trigonometric ratio.
You will use your knowledge of reference and rotation angles along with the CAST Rule to help you find these angles.

The first thing we will do is gain an understanding of the CAST Rule.

CAST Rule

Every primary trigonometric ratio (sinθ,cosθ,tanθ) is positive in two of the four quadrants and negative in the other two. The CAST rule is just a memory aid to help you remember the two quadrants where the primary trig. ratios are positive.(if you know where they are positive then automatically they are negative in the other 2 quadrants)



$$\sin\theta = \frac{y}{r}$$

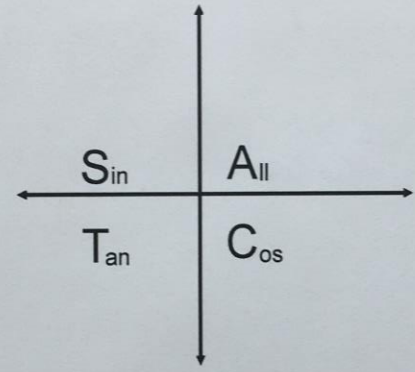
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

** r is always positive because it is the hypotenuse

Using quadrant 4 as an example you can see any point on the terminal arm in quadrant 4 would have a positive x and negative y. So because cosθ = x/r the sign of the cos ratio is positive while the other two ratios(sinθ,tanθ) would be negative because those ratios both contain y and the y value is negative in quadrant 4.

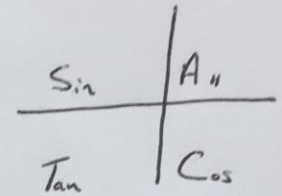
Cast Rule



Cosθ ratio is positive in quadrants 1 and 4 so negative 2 and 3.

Sinθ ratio is positive in quadrant 1 and 2 so negative in 3 and 4.

Tanθ ratio is positive in quadrant 1 and 3 so negative in 2 and 4.



What quadrant(s) am I in???

sin θ > 0 and tan θ < 0

Asking you where is sinθ positive and tanθ negative

sinθ is positive in quadrants 1 and 2 (CAST Rule)

tanθ is negative in quadrants 2 and 4 (CAST Rule)

So where is sinθ positive and tanθ negative

quadrant 2

Now we will put all trig. knowledge to work .

Finding multiple angles for any given trigonometric ratio...

Find all the angles for each trigonometric ratio for the restriction $0^\circ \le \theta \le 360^\circ$.

#1. $\cos\theta = 0.5736$, $0^\circ \le \theta \le 360^\circ$

① restriction says your answer must be between 0° and 360°

② restriction tells you how many answers you will have. There is an infinite number of angles that have this cos θ ratio, the restriction is saying to give only the angles between 0° and 360°.

* Positive Cos ratio

So from the CAST Rule you know cosθ is positive in quadrants 1 and 4.

So your answers must be from quadrants 1 and 4.

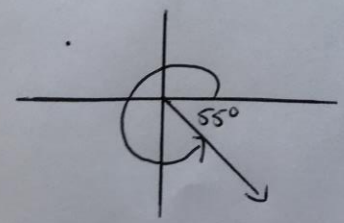
You need 2 answers.

First Answer

$\theta = \cos^{-1}.5736$

$\approx 55^\circ$ from quadrant 1

Second Answer Must come from quadrant 4
Use 55° as the reference angle and find rotation angle



$360^\circ - 55^\circ$

$= 305^\circ$

$55^\circ, 305^\circ$

Check answers

$\cos 55^\circ = .5736$

$\cos 305^\circ = .5736$

restriction answer must be between 0° and 360°

#2. $\sin\theta = 0.3420$, $0° \le \theta \le 360°$

positive ratio

Sin	All
Tan	Cos

Sin θ positive in quadrant 1 and 2 (CAST Rule)

Need 2 answers

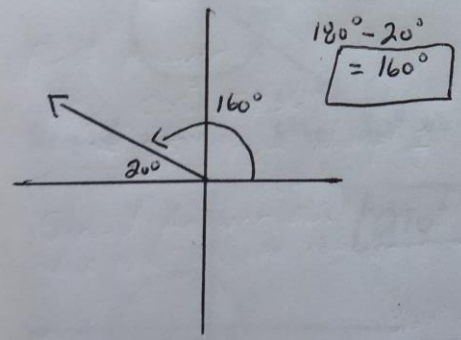
First Answers

$\theta = \sin^{-1} 0.3420$

$\approx 20°$

Second Answers

Use 20° as reference angle in 2nd quadrant



20°, 160°

check answers

sin 20° = .3420

sin 160° = .3420

positive angles only

#3. $\sin\theta = -0.5000$, $0° \le \theta \le 360°$

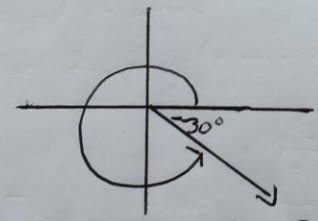
Sinθ is negative so the answers are in quadrant 3 and 4

So $\theta = \sin^{-1} -0.5000$

$= -30°$ *Be careful, $-30°$ is not one of your answers.

$\sin(-30°)$ is -0.5 but the restriction only wants positive answers between $0°$ and $360°$

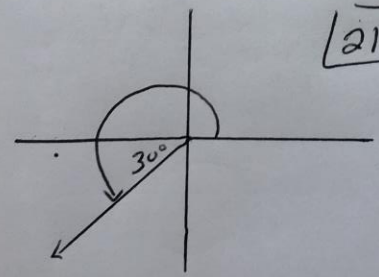
Find the positive coterminal of $-30°$



330°

$-30° + 360° = 330°$

Second answer Use $30°$ as reference angle



210°

210°, 330°

check

$\sin 210° = -0.5$

$\sin 330° = -0.5$